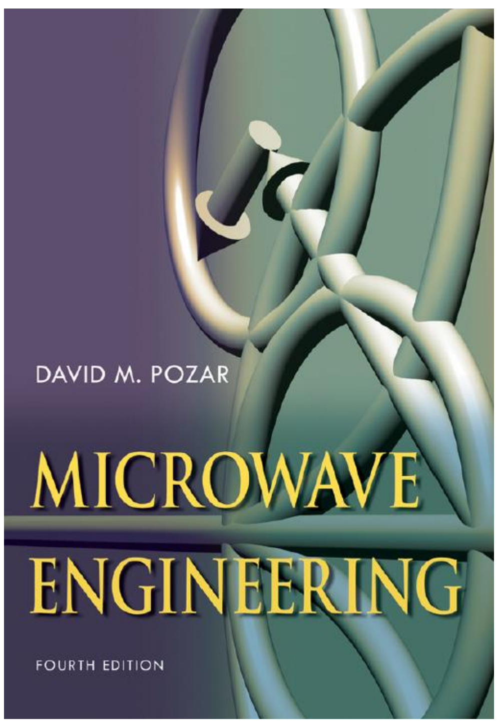
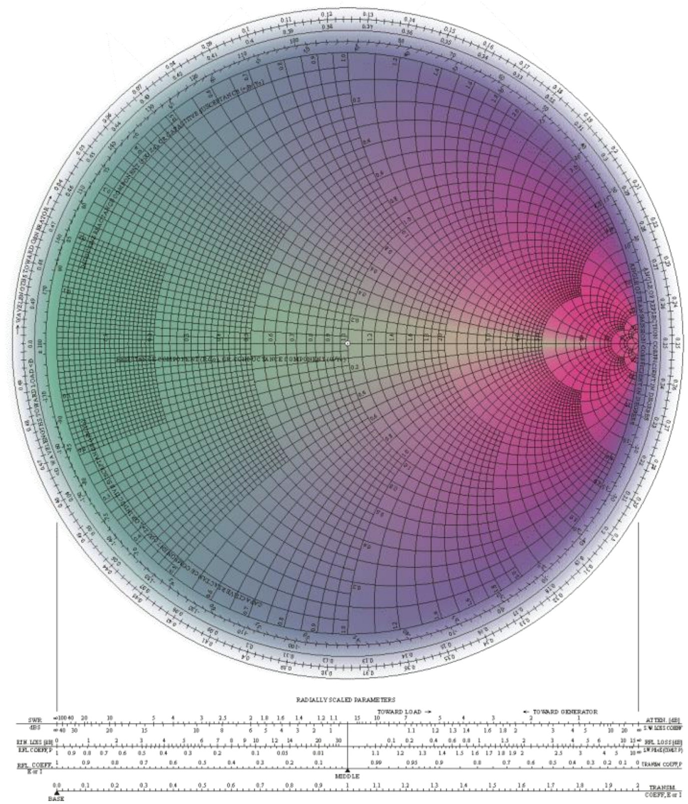
ECE 344
Microwave Fundamentals

Spring 2017

Lectre 03:
Smith Charts





ILOS

- Smith Chart
  - History.
  - Construction & Transformation.
  - Scales.
- Examples of how to use Smith Chart to calculate:
  - Reflection Coefficient.
  - Standing wave ratio.
  - Input impedance.
  - Location of the first maximum and minimum.
  - Admittance.

Introduction

- The Smith chart developed in 1939 by P. Smith, in the Bell Telephone laboratories.
- It is a graphical procedure for solving impedance transformation problems to reduce the computational effort required.
- In practice, high-frequency circuits often contain two or more transmission lines interspersed with series and shunt elements.
- The Smith chart technique can significantly reduce the numerical and algebraic manipulations required to solve such problems.

Introduction

The Smith chart is a specially constructed impedance/admittance diagram used in solving TL problems. Usefull characteristics of the Smith chart are:

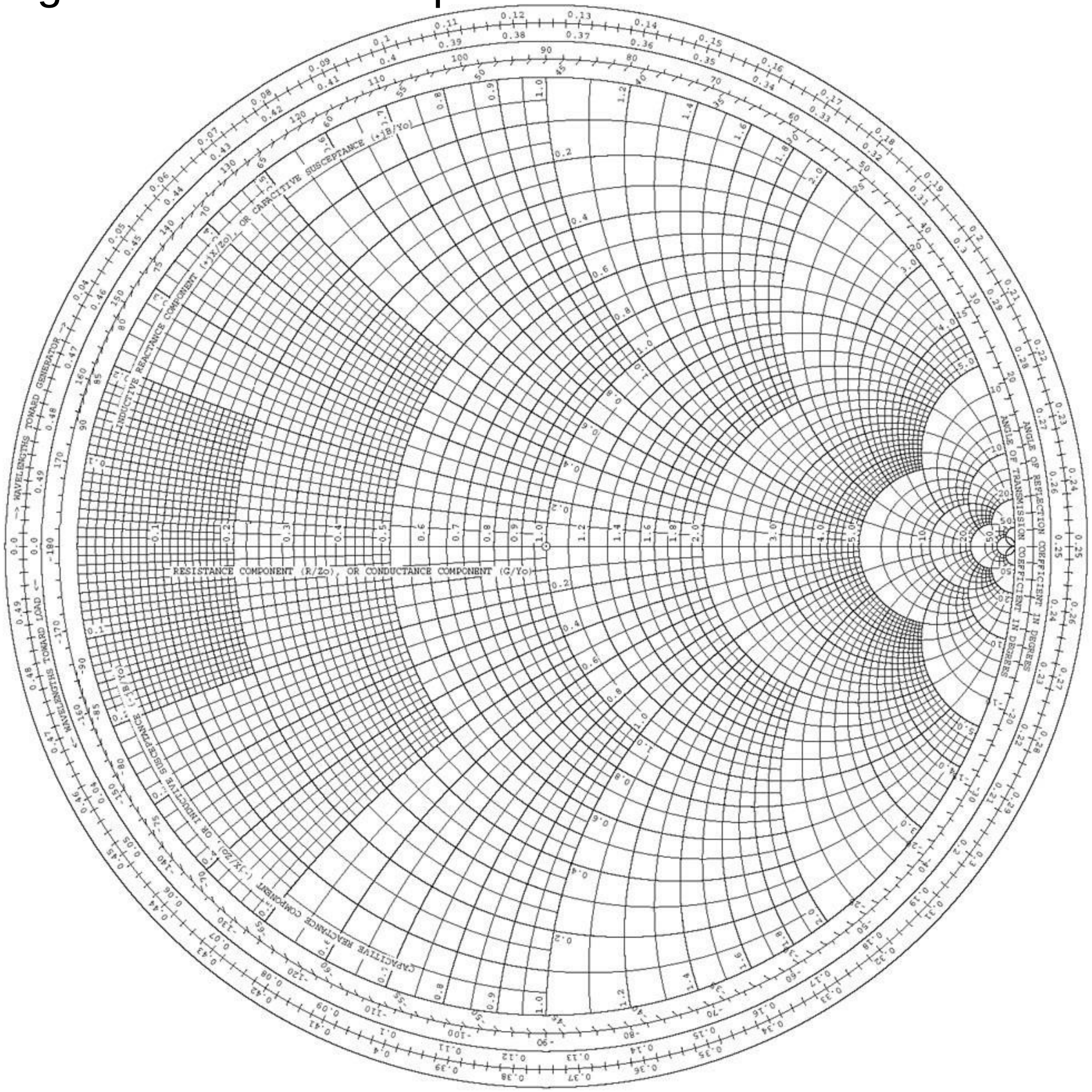
a) all possible values of impedance and admittance can be plotted on the chart,

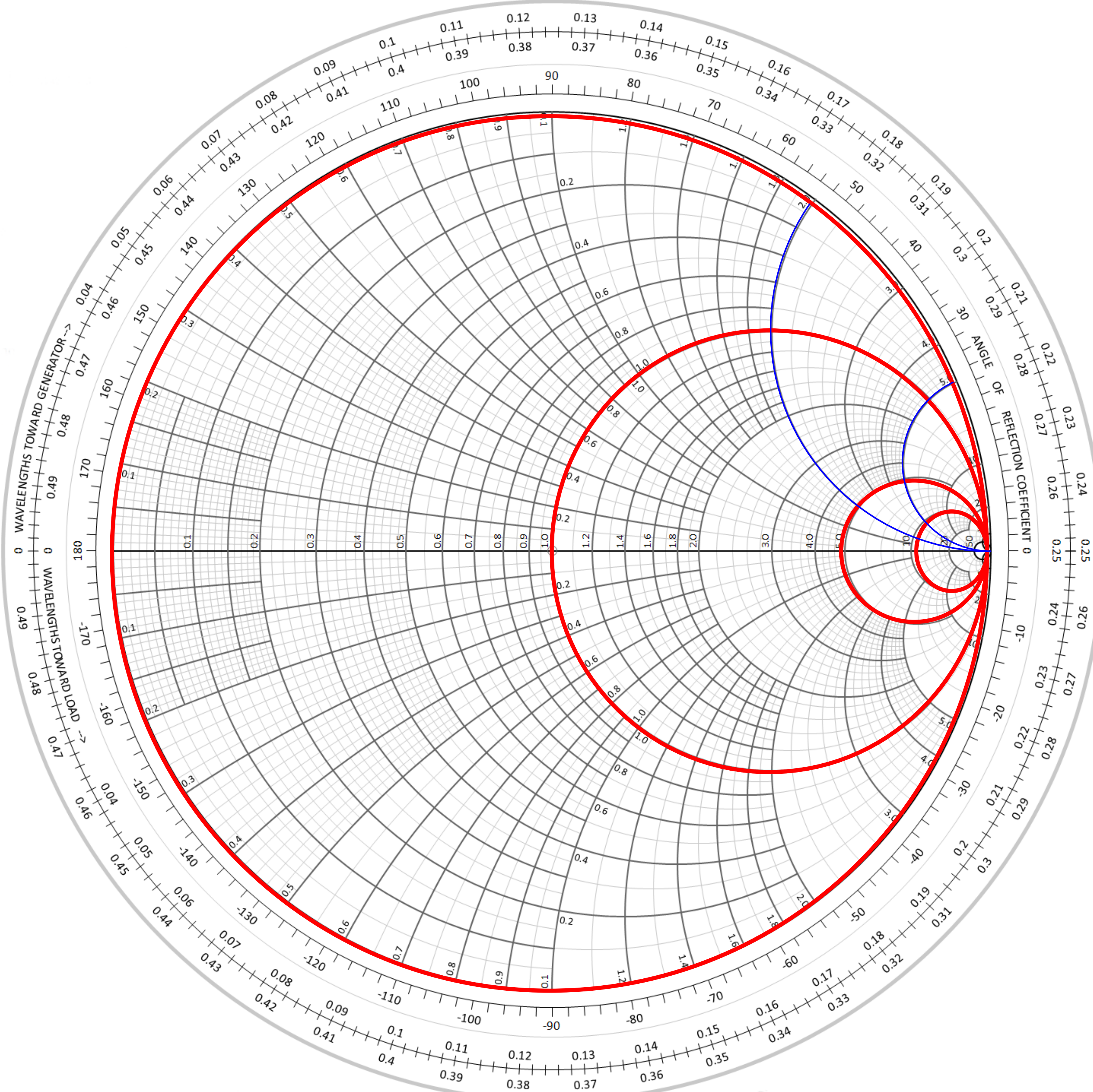
b) an easy method for converting impedance to admitance (and vice versa) is available,

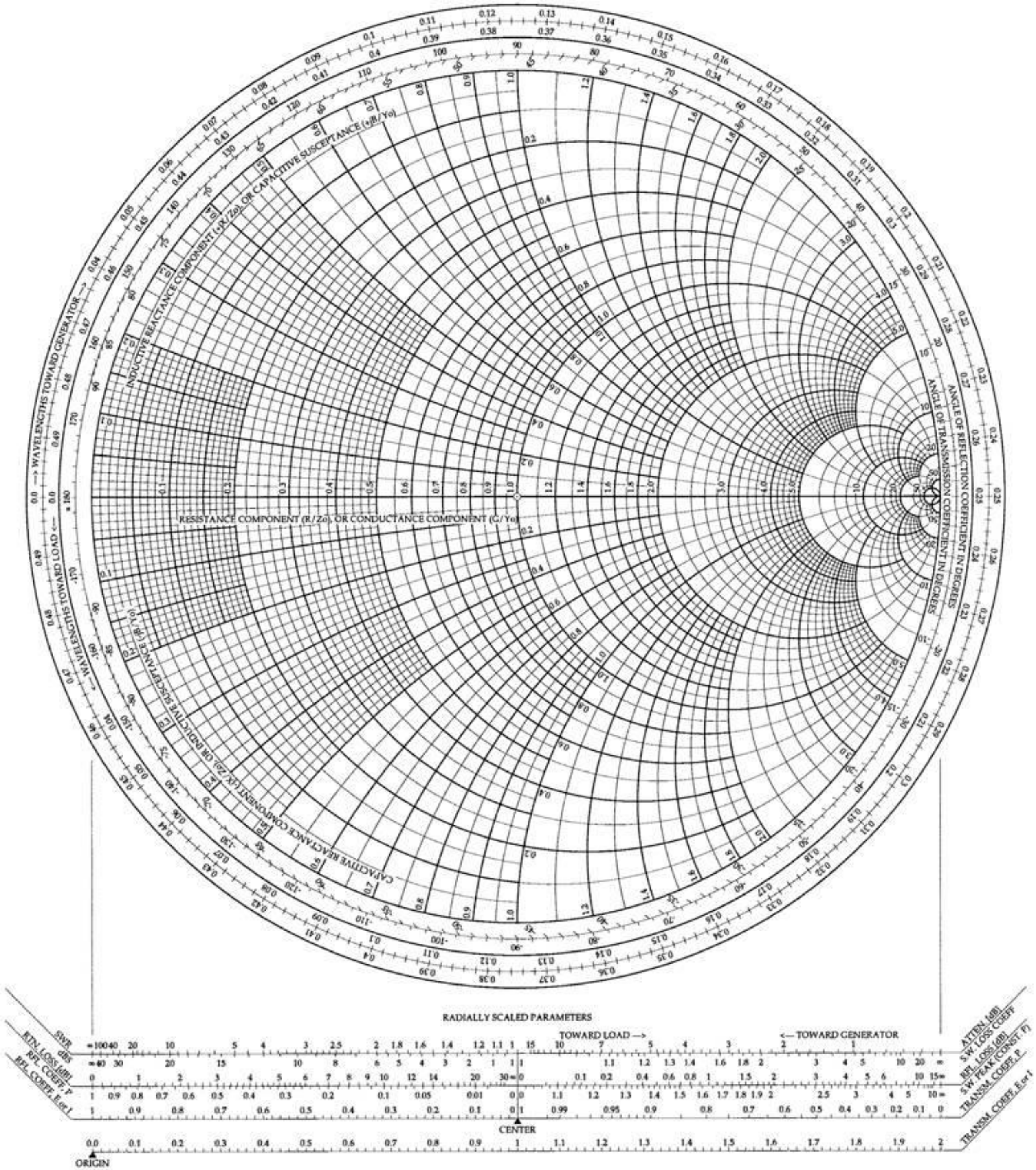
c) the Smith chart provides a simple graphical method for determining the impedance transformation due to a length of TL

Introduction

The Smith chart, shown in the figure below, is a graphical aid that can be very useful for solving transmission line problems.

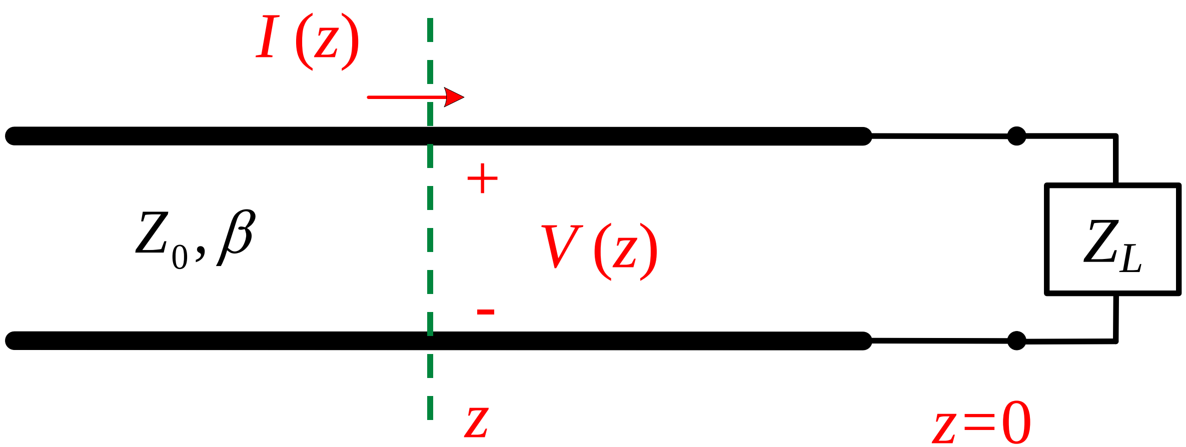



WAVELENGTHS TOWARD GENERATOR —>
WAVELENGTHS TOWARD LOAD —>
ANGLE OF REFLECTION COEFFICIENT



WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF. P
RFL. COEFF. E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF. P
TRANSM. COEFF. E or I
CENTER
ORIGIN

Generalized Reflection Coefficient



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Recall:

$$V(z) = V_0^+ e^{-\gamma z}\left(1+\Gamma_L e^{+2\gamma z}\right) = V_0^+ e^{-\gamma z}\left(1+\Gamma(z)\right)$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z}\left(1-\Gamma_L e^{+2\gamma z}\right) = \frac{V_0^+}{Z_0} e^{-\gamma z}\left(1-\Gamma(z)\right)$$

$$Z(z) = \frac{V(z)}{I(z)} = Z_0\left(\frac{1+\Gamma_L e^{+2\gamma z}}{1-\Gamma_L e^{+2\gamma z}}\right) = Z_0\left(\frac{1+\Gamma(z)}{1-\Gamma(z)}\right)$$

Generalized reflection Coefficient: $\Gamma(z) = \Gamma_L \, e^{+2\gamma z}$

Generalized Reflection Coefficient (cont.)

$$\Gamma(z) = \Gamma_L \, e^{+2\gamma z}$$
$$= |\Gamma_L| \, e^{j\phi_L} \, e^{+2\gamma z}$$
$$= \Gamma_R(z) + j\Gamma_I(z)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

For

$$\mathrm{Re}\{Z_L\} \geq 0$$
$$\Rightarrow |\Gamma_L| \leq 1$$

Lossless transmission line ($\alpha = 0$)

$$\Gamma(z) = |\Gamma_L| e^{j(\phi_L + 2\beta z)}$$

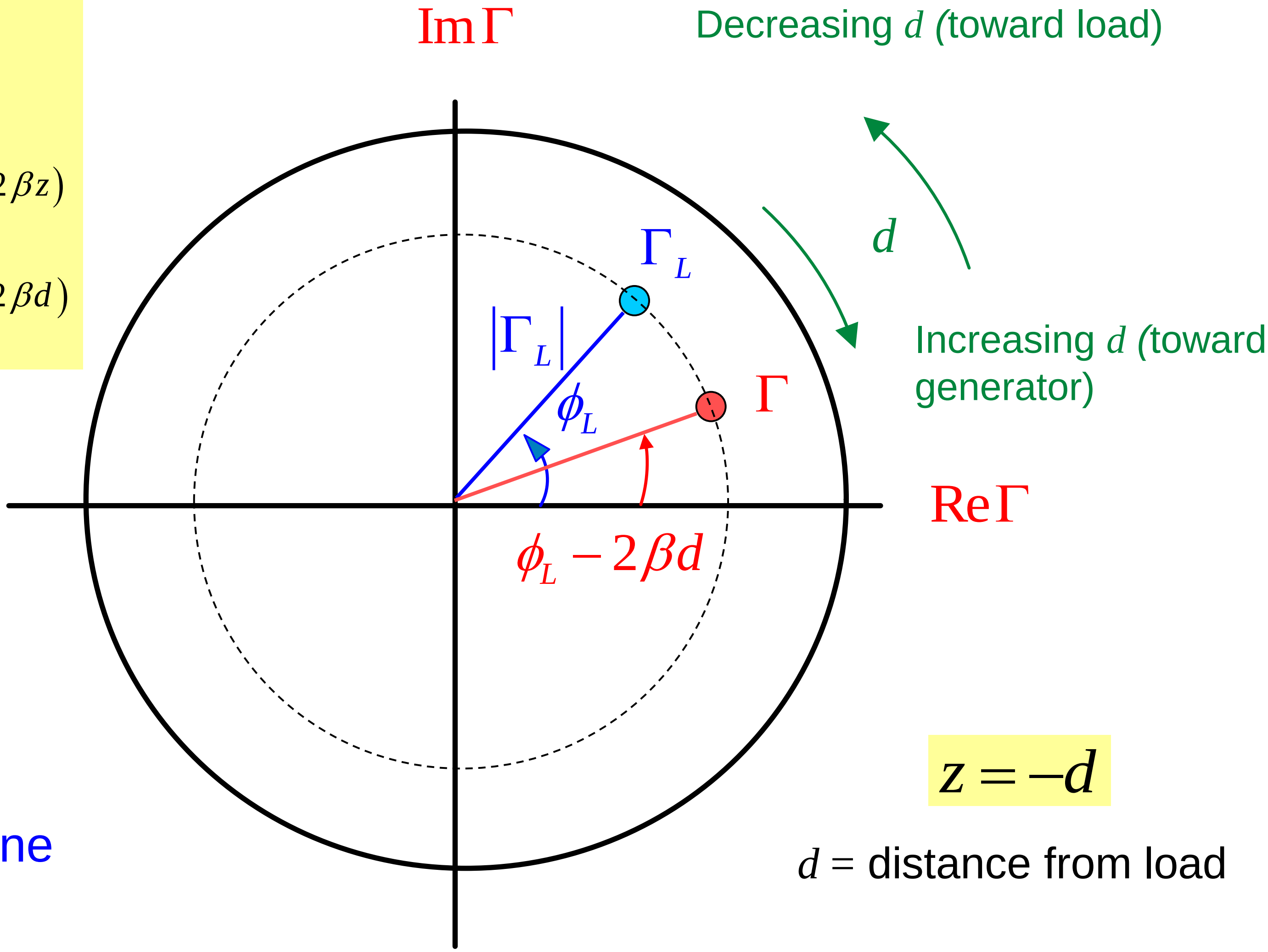
Proof:

$$\Gamma_L = \frac{(R_L + jX_L) - Z_0}{(R_L + jX_L) + Z_0}$$
$$= \frac{(R_L - Z_0) + jX_L}{(R_L + Z_0) + jX_L}$$

$$\Rightarrow |\Gamma_L|^2 = \frac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2}$$

Complex Γ Plane

$$\Gamma = \Gamma(z)$$
$$= \Gamma_R + j\Gamma_I$$
$$= \Gamma_L e^{j(+2\beta z)}$$
$$= |\Gamma_L| e^{j(\phi_L + 2\beta z)}$$
$$= |\Gamma_L| e^{j(\phi_L - 2\beta d)}$$



$$z = -d$$

Lossless line

d = distance from load

Smith Chart Construction

It is a transformation from the impedance graph, to a reflection coefficient graph.

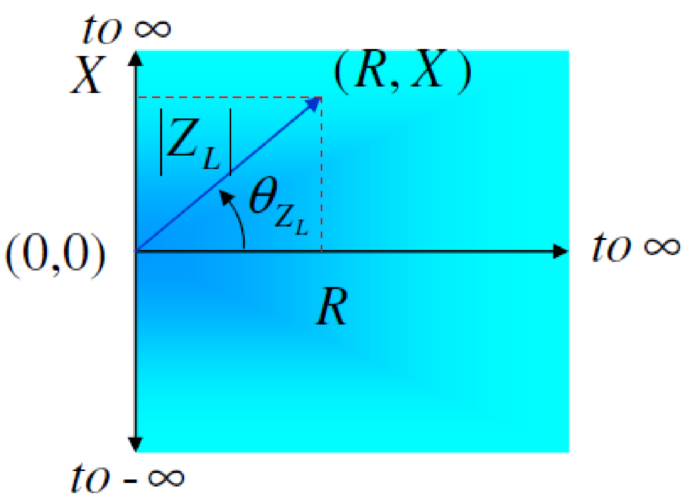
$$Z_L = R + jX = |Z_L| e^{j\theta_{Z_L}}$$

Where $R = \Re e(Z_L)$

$X = \Im m(Z_L)$

Note $0 \le R < \infty$

$-\infty < X < \infty$



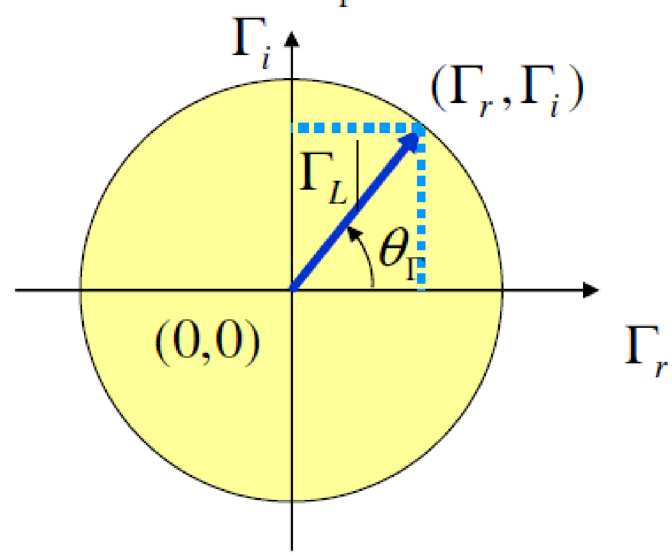
$$\Gamma_L = |\Gamma_L| e^{j\theta_\Gamma} = \Gamma_r + j\Gamma_i$$

Where $\Gamma_r = \Re e(\Gamma)$

$\Gamma_i = \Im m(\Gamma)$

Note $0 \le |\Gamma_L| < 1$

$-180 \le \theta_\Gamma < 180$



Smith Chart Construction

The relation between both domains are given by

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

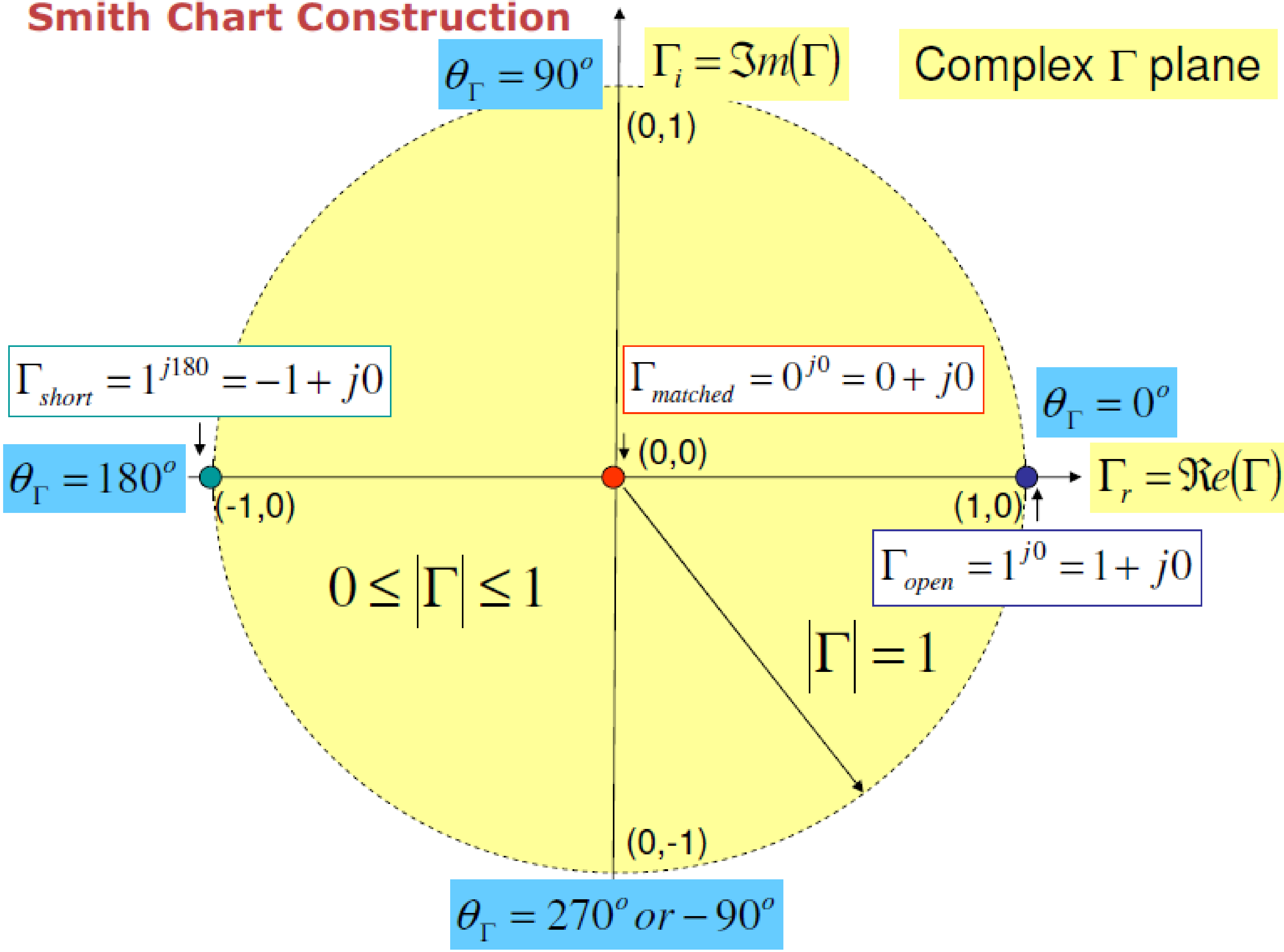
Instead of having separate Smith Charts for TL with different characteristic impedances such as Z_o=50, 75, 100 Ω, etc; only one normalized Smith chart do the job!

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1} = \frac{z_L - 1}{z_L + 1}$$

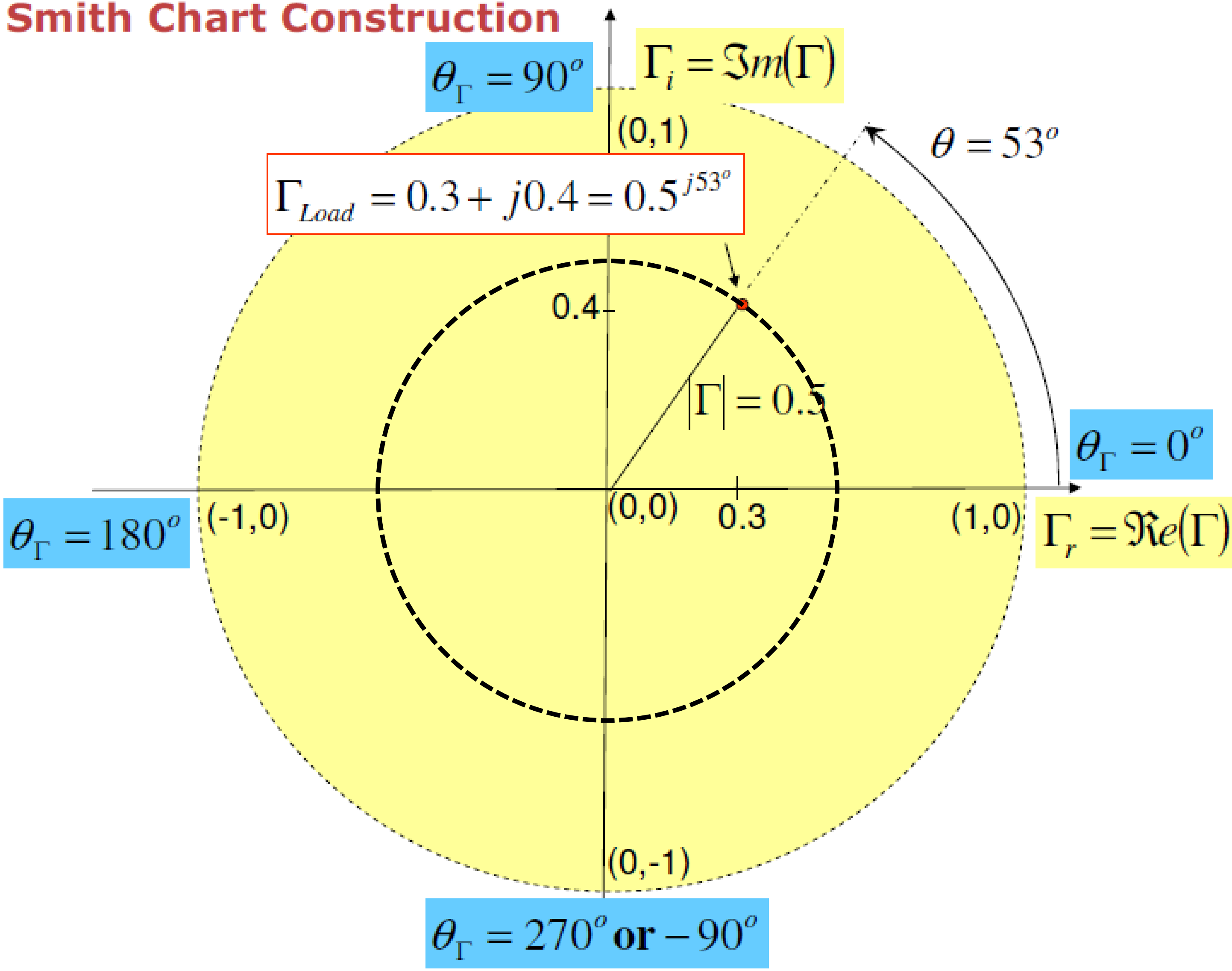
,Where

$$z_L = \frac{Z_L}{Z_o} = \frac{R}{Z_o} + j\frac{X}{Z_o} = r + jx$$

Smith Chart Construction



Smith Chart Construction



Smith Chart

- The transformation

$$\Gamma_L = \frac{z_L - 1}{z_L + 1} = \Gamma_r + j\Gamma_i \quad \text{Rearrange} \quad z_L = \frac{1+\Gamma_L}{1-\Gamma_L} = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i} = r + jx$$

Equating Both Sides

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} \quad \& \quad x = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Rearrange

$$\left[\Gamma_r - \frac{r}{1+r}\right]^2 + \Gamma_i^2 = \left[\frac{1}{1+r}\right]^2$$

Circle Equations

$$[\Gamma_r - 1]^2 + \left[\Gamma_i - \frac{1}{x}\right]^2 = \left[\frac{1}{x}\right]^2$$

$$(x - x_0)^2 + (y - y_0)^2 = a^2$$

Convert a constant **r** line from **Z** to Γ

$$radius = 1/(1+r)$$

$$center = (r/(1+r), 0)$$

Convert a constant **x** line from **Z** to Γ

$$radius = 1/x$$

$$center = (1, 1/x)$$

Let, $\Gamma_L = \Gamma_r + j\Gamma_i$

And $\bar{z}_L = r_L + jx_L$, then $r_L + jx_L = \dfrac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}$

$$r_L + jx_L = \frac{\left[(1+\Gamma_r) + j\Gamma_i\right]\left[(1-\Gamma_r) + j\Gamma_i\right]}{(1-\Gamma_r)^2 + \Gamma_i^2} = \frac{1-\Gamma_r^2 - \Gamma_i^2 + j2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$r_L = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} \qquad x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$r_L - 2\Gamma_r r_L + \Gamma_r^2 r_L + \Gamma_i^2 r_L = 1 - \Gamma_r^2 - \Gamma_i^2$$

$$\Gamma_r^2 (r_L + 1) + \Gamma_i^2 (r_L + 1) - 2\Gamma_r r_L = 1 - r_L$$

$$\Gamma_r^2 + \Gamma_i^2 - \frac{2r_L}{r_L + 1}\Gamma_r = \frac{1 - r_L}{1 + r_L}$$

$$\left(\Gamma_r - \frac{r_L}{r_L + 1}\right)^2 - \frac{r_L^2}{(r_L + 1)^2} + \Gamma_i^2 = \frac{1 - r_L}{1 + r_L}$$

$$\left(\Gamma_r - \frac{r_L}{r_L + 1}\right)^2 + \Gamma_i^2 = \frac{1 - r_L}{1 + r_L} + \frac{r^2{}_L}{(r_L + 1)^2} = \frac{1 - r^2{}_L + r^2{}_L}{(1 + r_L)^2}$$

$$\left(\Gamma_r - \frac{r_L}{r_L + 1}\right)^2 + \Gamma_i^2 = \frac{1}{(1 + r_L)^2}$$

Resistance circles

$$\Updownarrow$$

$$(x - x_0)^2 + (y - y_0)^2 = a^2$$

$$\Rightarrow$$

$$radius = 1/(1 + r)$$

$$center = (r/(1 + r), 0)$$

Similarly,

$$x_L - 2\Gamma_r x_L + \Gamma_r^2 x_L + \Gamma_i^2 x_L = 2\Gamma_i$$

$$\Gamma_r^2 - 2\Gamma_r + 1 + \Gamma_i^2 - 2\frac{\Gamma_i}{x_L} = 0$$

$$\Gamma_r^2 - 2\Gamma_r - 1 + \Gamma_i^2 - 2\frac{\Gamma_i}{x_L} = 0$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \frac{1}{x_L^2}$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

Reactance circles

$$\updownarrow$$

$$(x - x_0)^2 + (y - y_0)^2 = a^2 \quad \Rightarrow$$

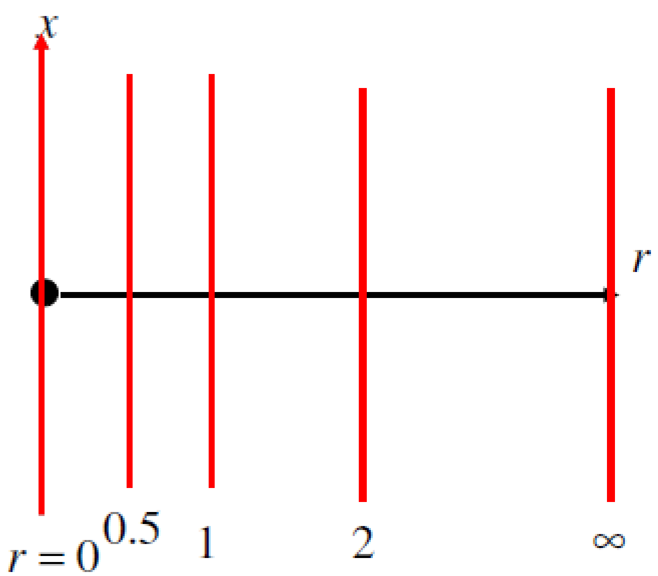
$$radius = 1/x$$

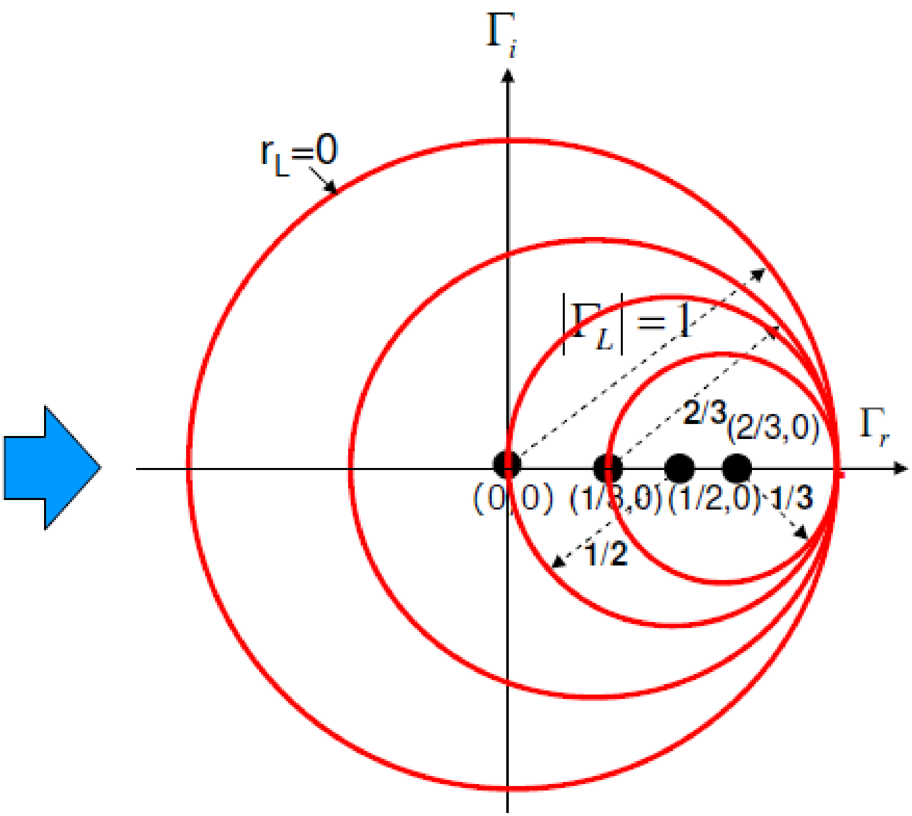
$$center = (1, 1/x)$$

$$center = \left(r/(1+r), 0 \right) \qquad radius = 1/(1+r)$$

Transforming "r"

r	Radius	Center
0	1	(0,0)
1/2	2/3	(1/3,0)
1	1/2	(1/2,0)
2	1/3	(2/3,0)
∞	0	(1,0)





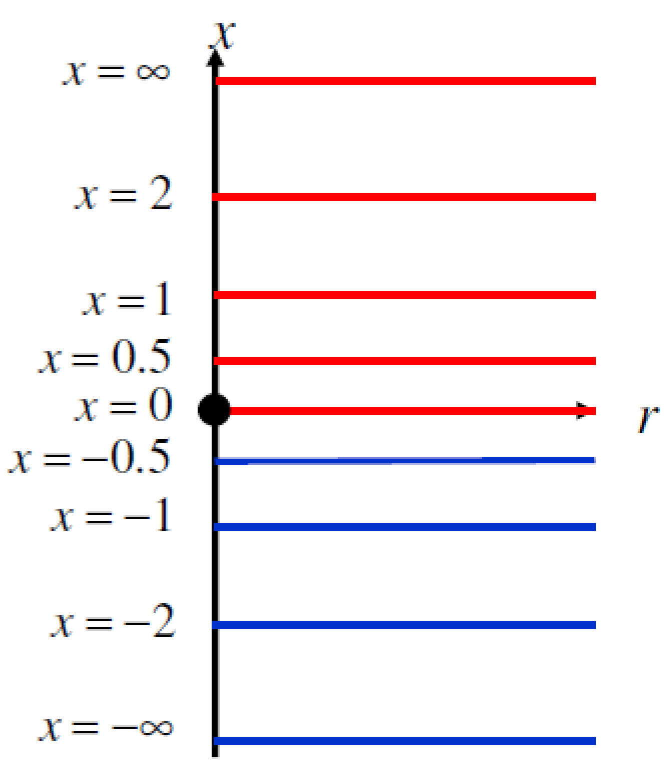
Transforming "x"

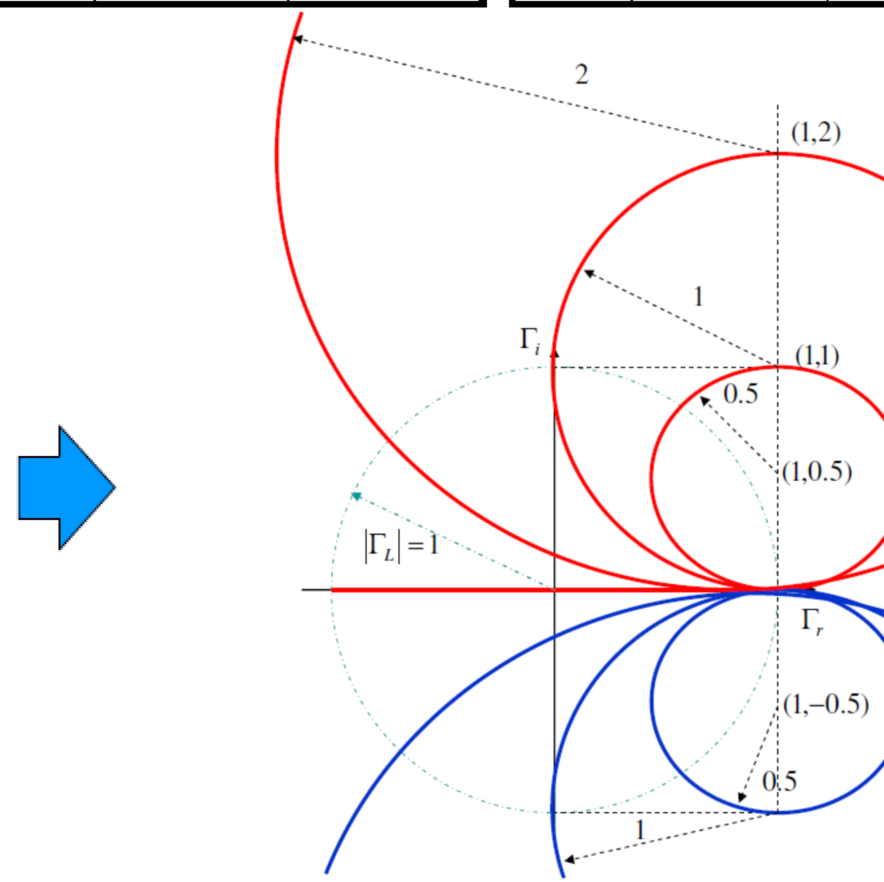
$$center = (1, 1/x)$$

$$radius = 1/|x|$$

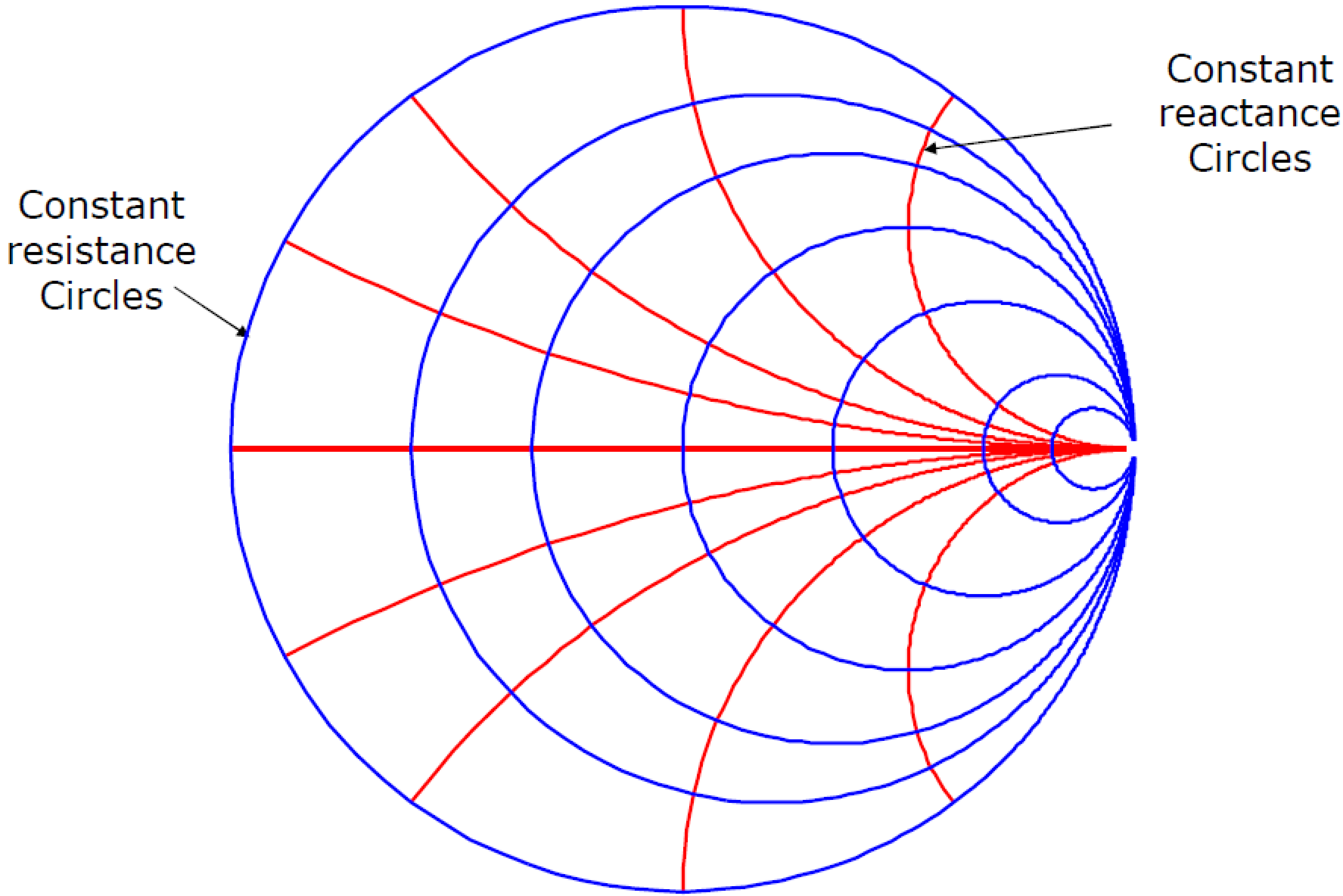
x	Radius	Center
0	∞	(1,∞)
0.5	2	(1,2)
1	1	(1,1)
2	0.5	(1,0.5)
∞	0	(1,0)

x	Radius	Center
0	∞	(1,-∞)
-0.5	2	(1,-2)
-1	1	(1,-1)
-2	0.5	(1,-0.5)
-∞	0	(1,0)

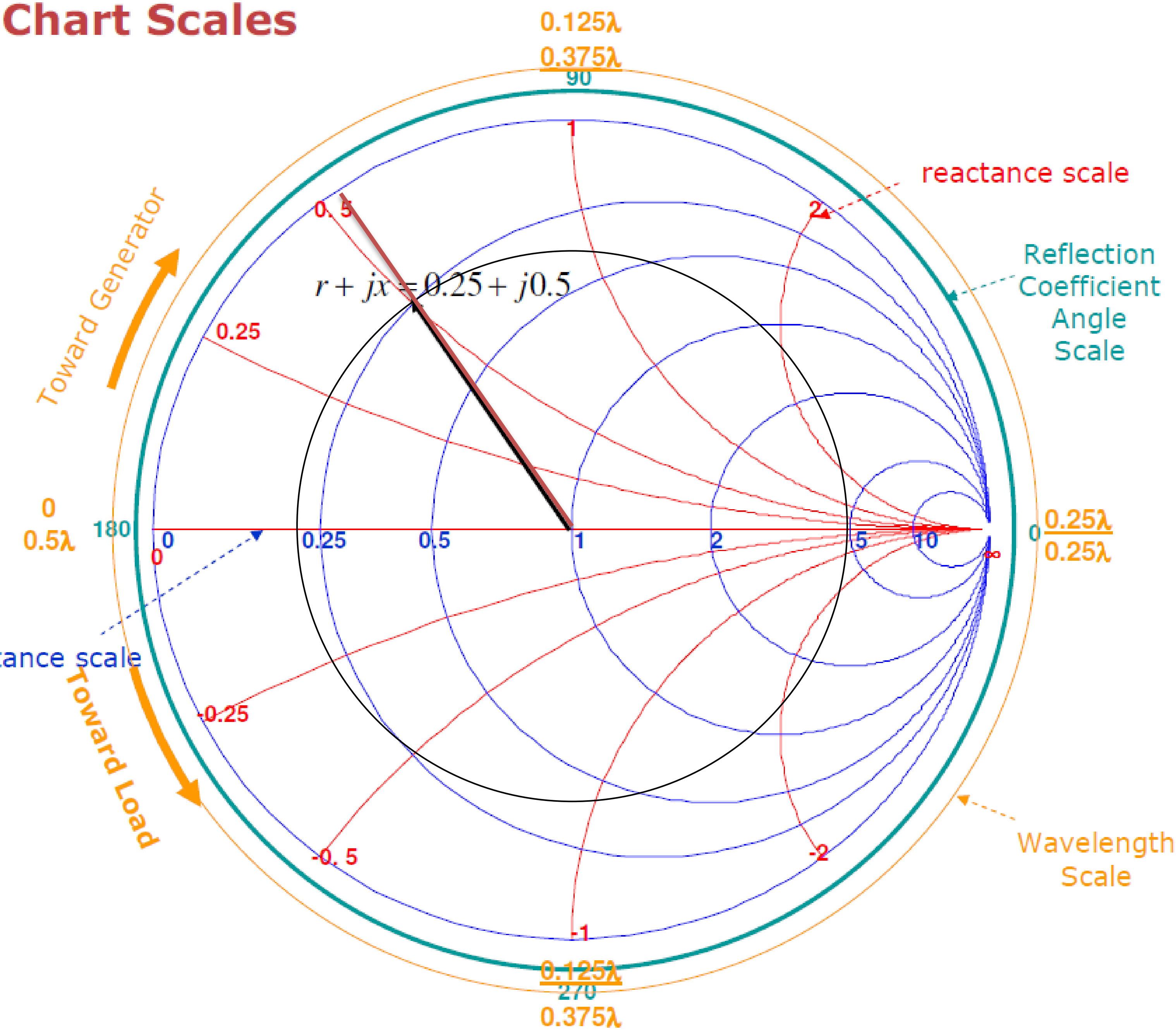




Smith Chart Construction



Smith Chart Scales



Important notes about smith chart

Some Points:

i) The angle of reflection coefficient scale is the innermost of the three scales on the periphery of the chart.

ii) The two outermost scales allow us to perform the Γ transformations (and hence the Z transformation) without calculating $2\beta d$. The scales are directly in fractions of wavelength. Note that one complete rotation around the chart is a half wavelength.

iii) The Smith chart can also be used with normalized admittance coordinates. For any $\bar{z}$, its equivalent normalized admittance $\left(\bar{Y} = \frac{Y}{Y_0}\right)$ is 180^0 away on the same $|\Gamma|$ circle.

The following comments should be kept in mind:

1) The $\bar{Y} = 0$ corresponds to an open circuit, while the $\bar{Y} = \infty$ corresponds to a short circuit.

2) The resistance coordinates become conductance and the reactance coordinates become susceptance coordinates.

3) There are two wavelength scales on the periphery of the chart. One is labeled as Wavelengths toward Generator and the other Wavelengths toward the Load. It is used to find determine the impedance at a point nearer the input than the known impedance.

The clockwise rotation is referred to as mowing toward the generator and counterclockwise rotation is referred to as mowing toward the load.

Important notes about smith chart

4) Each $|\Gamma|$ circle intersects the real axis at two points, the point on the positive-real axis and the point on the negative real axis. $X_L = 0$ along the real axis and $Z_L = R_L$.

$$\Gamma = \frac{r_L - 1}{r_L + 1} \qquad S = \frac{1+\Gamma}{1-\Gamma} = \frac{1 + \dfrac{r_L - 1}{r_L + 1}}{1 - \dfrac{r_L - 1}{r_L + 1}} = \frac{r_L}{1}$$

$$S = r_L$$

This shows that the value of the r -circle corresponding to $|\Gamma|$ passing through the positive real axis is equal to the standing wave ration.

$$z_L = \frac{Z_L}{Z_0} = 0.25 + j0.5$$

Γ circle

$|\Gamma_1| = |\Gamma_2|$

for $z_{L1} = r_{L1} + jx_{L1}$

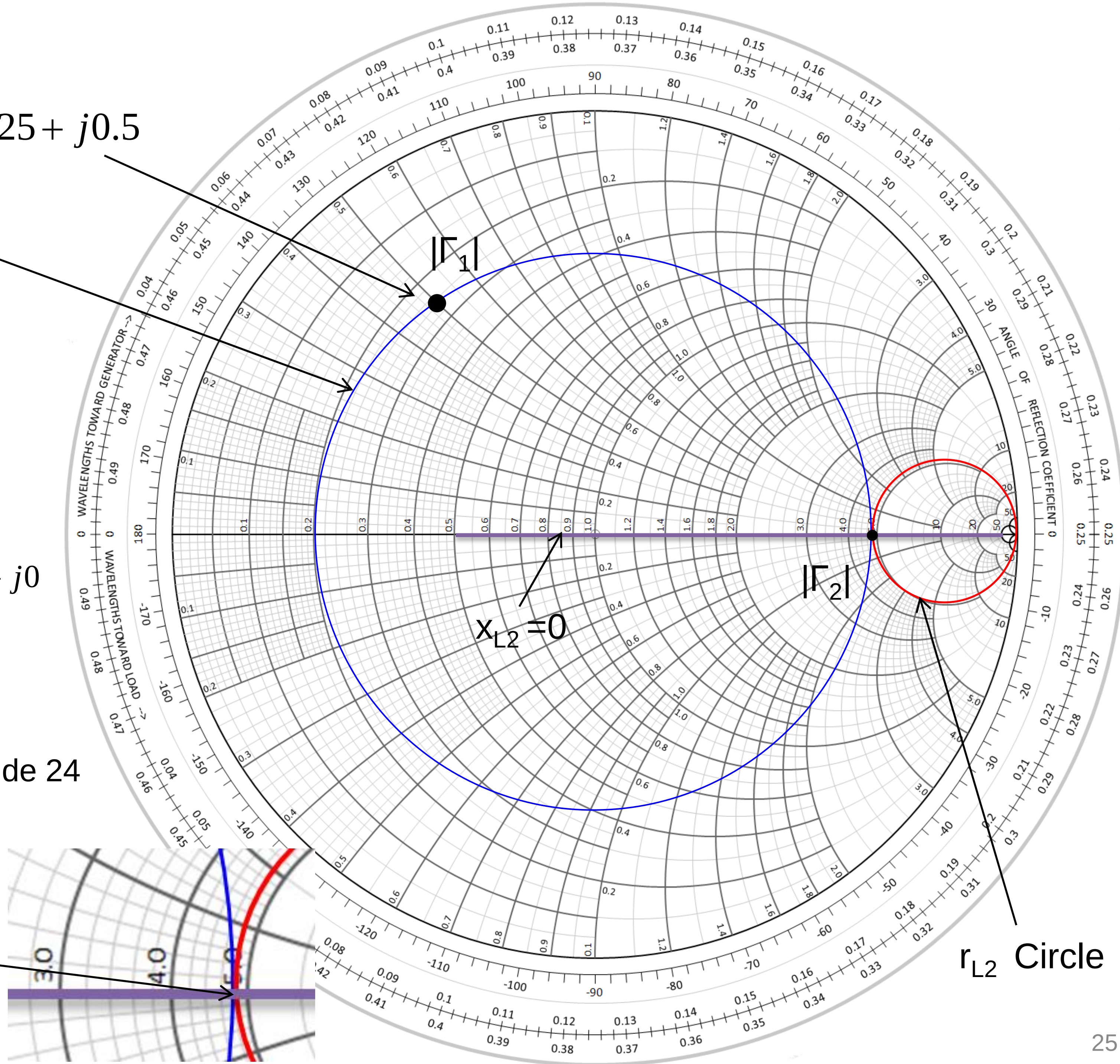
for $z_{L2} = r_{L2} + j0$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

As proofed from slide 24

$$S = \frac{1+|\Gamma_2|}{1-|\Gamma_2|} = r_{L2}$$

$$\therefore S = 4.88$$



Solutions steps

1) find the normalized load impedance $$z_L = \frac{Z_L}{Z_0} = r_L + jx_L$$

2) Locate r_L and x_L circles on the smith chart.

3) intersection of r_L and x_L circles will give you a point z_L

4) draw the Γ_L circle to pass through the point z_L, where the center of Γ_L circle is the center of the smith chart.

5) To calculate Γ_L , use you ruler to measure the distance 0 ➔ z_L then project this distance on the linear scale of reflection coefficient to calculate Γ_L

6) the intersection of the Γ_L circle with the horizontal line of the smith chart (Γ_r axis) gives you the value of SWR as explained in slide 24.

EXAMPLE 2.2 BASIC SMITH CHART OPERATIONS

A load impedance of **40 + j70 Ω** terminates a 100 **Ω** transmission line that is 0.3λ long. Find

1- the reflection coefficient at the load,

2- the reflection coefficient at the input to the line,

3- the input impedance,

4- the standing wave ratio on the line, and

5- the return loss.

Solution

The normalized load impedance is

$$z_L = \frac{Z_L}{Z_0} = 0.4 + j0.7$$

$z_L = 0.4 + j0.7$
$= r_L + jx_L$

the reflection coefficient at the load

$$|\Gamma_L| = \frac{ab}{ac} = \frac{1.69}{2.86} = 0.59$$

$$\theta_{\Gamma,L} = 104^\circ$$

The reflection coefficient at the input

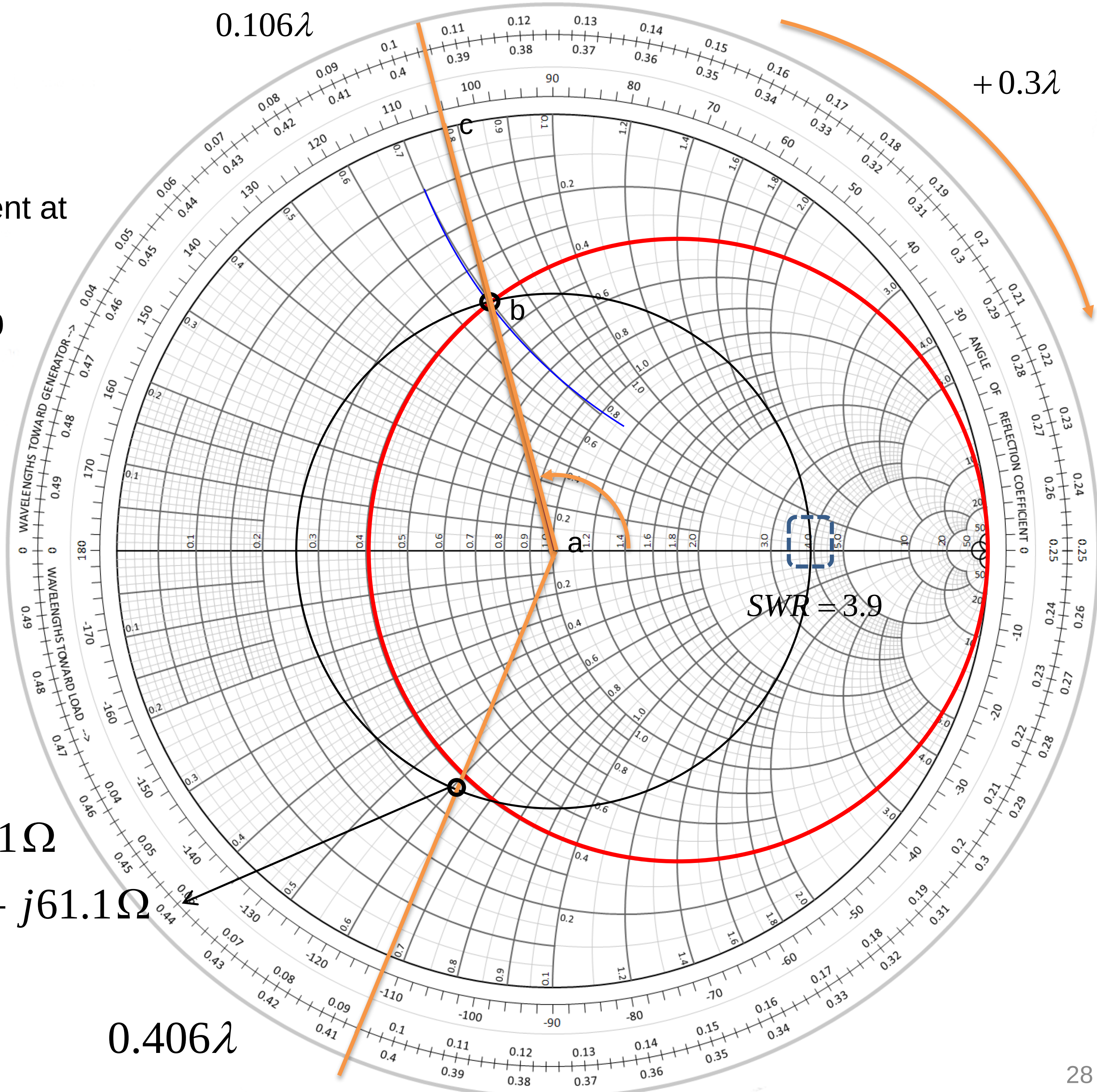
$$|\Gamma_{in}| = 0.59$$

$$\theta_{\Gamma,in} = 248^\circ$$

RL = 4.6 dB.

$$z_{in} = 0.365 - j0.611\Omega$$

$$Z_{in} = Z_0 z_{in} = 36.5 - j61.1\Omega$$



Example 1

(Calculating Reflection Coefficient)

If Z_L=100-j50 Ω, & Z_o=50 Ω

– Find Γ_L?

Ans:.

1st Normalize the Value of Z_L

$$z_L = (100 - j50)/50$$

$$z_L = 2 - j1$$

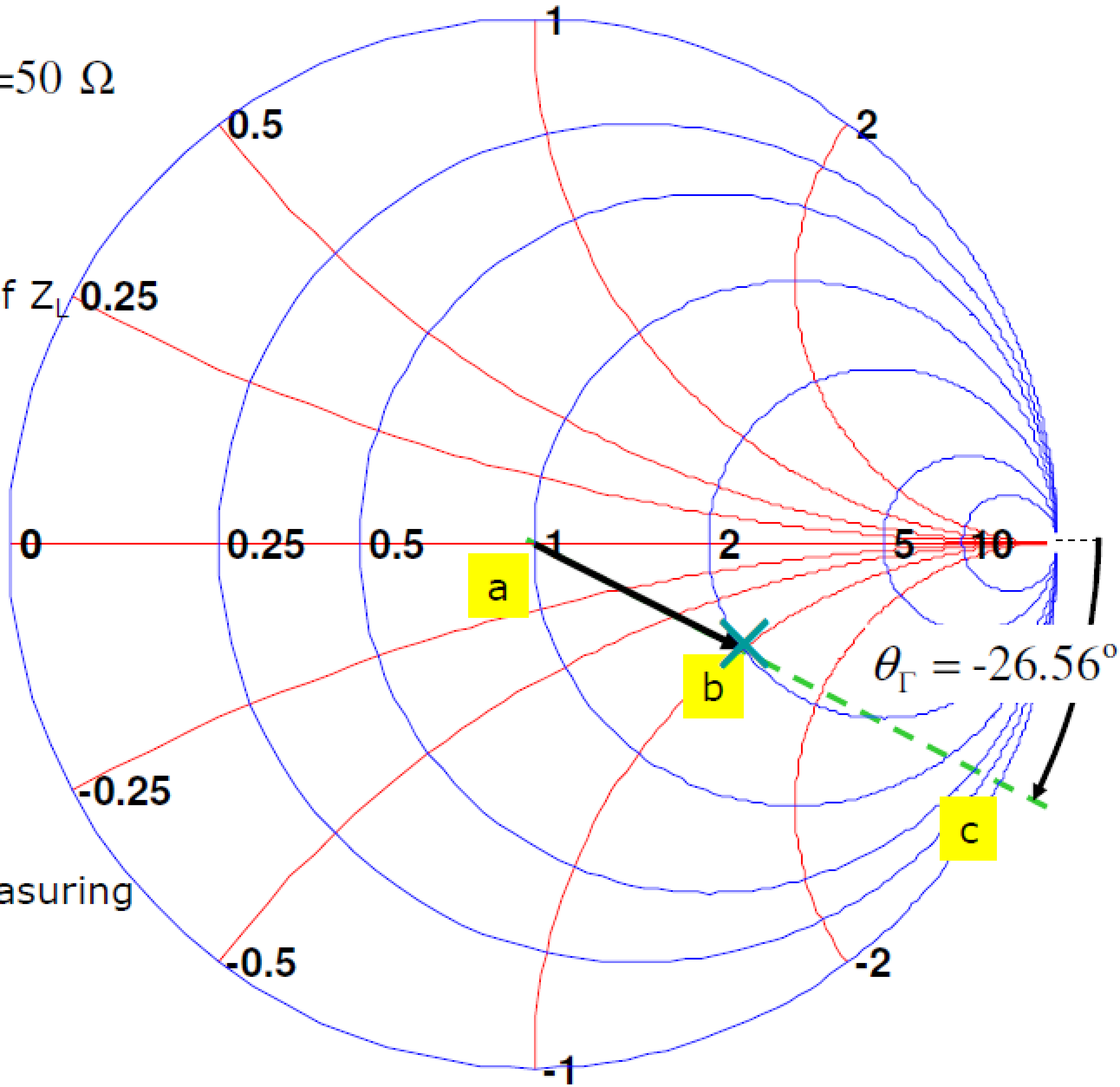
2nd Locate Z_L on SC

3rd Draw a line from the center of SC passing by Z_L until the edge of SC

4th Use the angle scale to determine the value of θ_L.

5th Calculate $|\Gamma_L|$, by measuring

$$|\Gamma_L| = \frac{|\vec{r}_{ab}|(cm)}{|\vec{r}_{ac}|(cm)} = 0.447$$



Example 1 (cont.)

(Calculating input impedance)

- The reflection coefficient at any location (z)

$$\Gamma(z) = \frac{V_o^- e^{j\beta z}}{V_o^- e^{-j\beta z}} = \Gamma_L e^{2j\beta z}$$

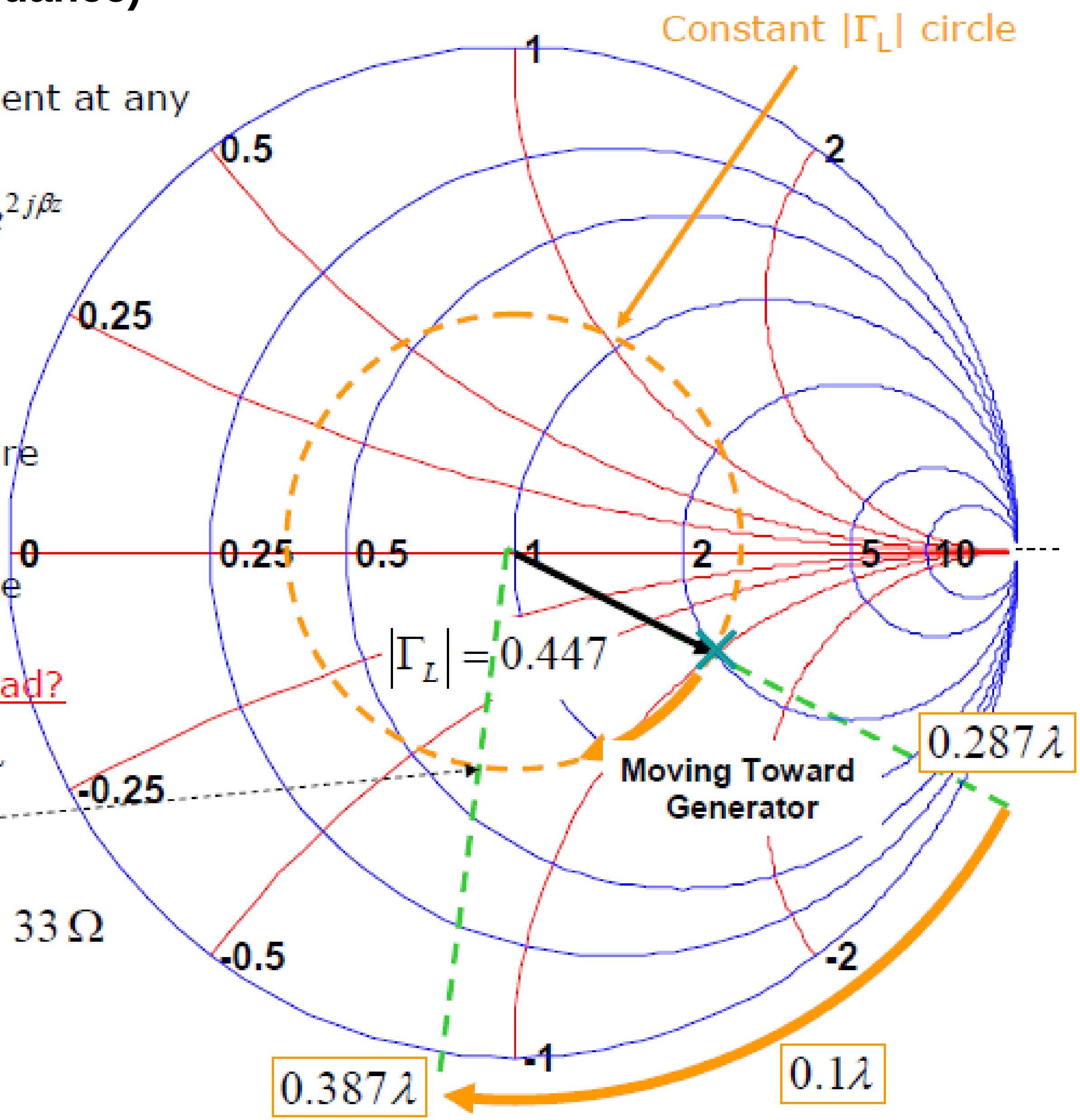
Accordingly, moving along the <u>same</u> transmission line will always mean that we are moving on a constant circle with radius equal $|\Gamma_L|$, and only the phase will change.

Find Z_{in} at 0.1λ from load?

New location is 0.387λ

$$z_{in} = 0.6 - j\,0.66$$

$$Z_{in} = z_{in} \times Z_o = 30 - j\,33\,\Omega$$



Example 1 (cont.)

(Calculating location of Vmin and Vmax)

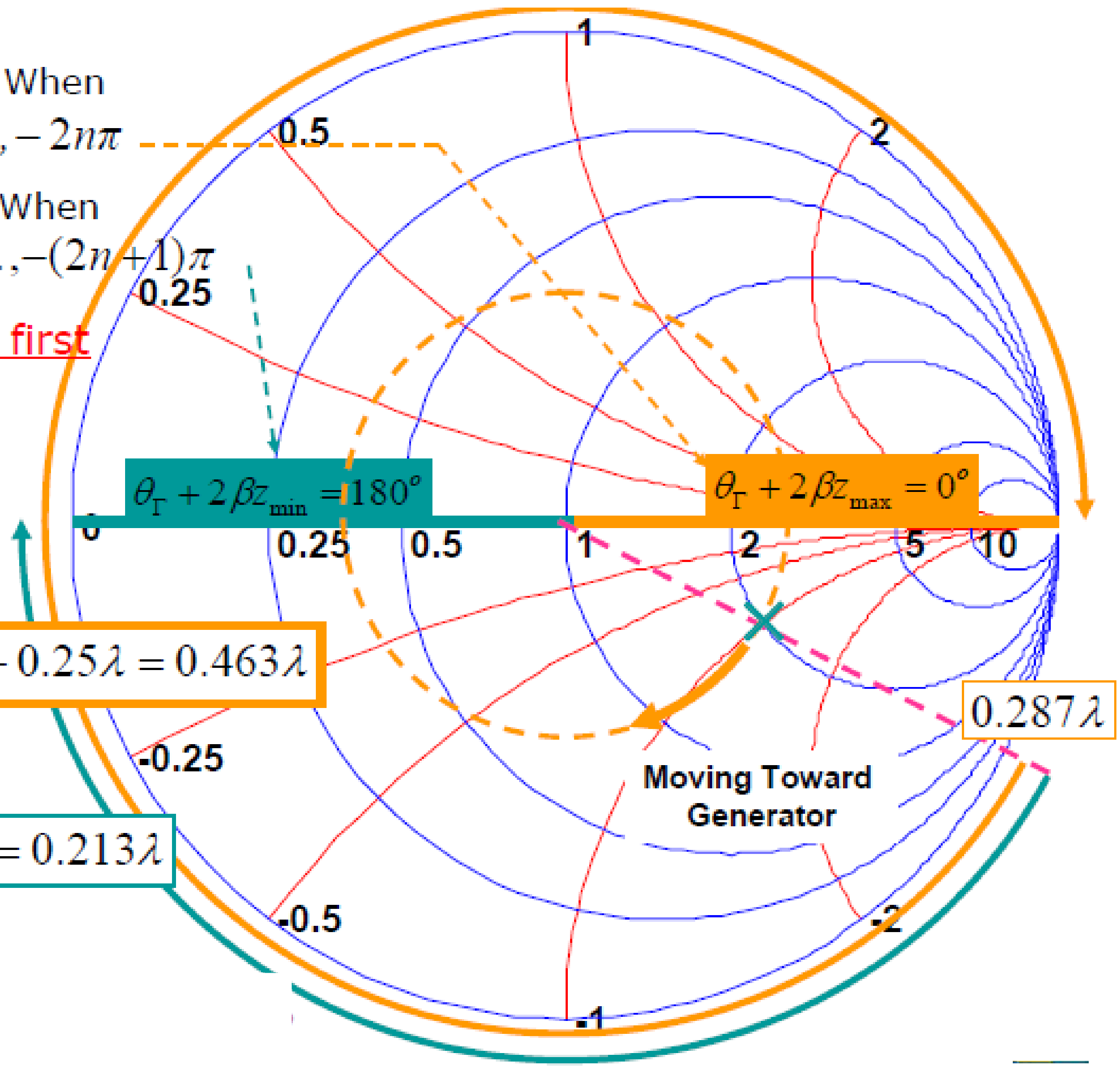
- Maximum Occurs When

$$\theta_\Gamma + 2\beta z = 0, -2\pi, \ldots, -2n\pi$$

Minimum Occurs When

$$\theta_\Gamma + 2\beta z = -\pi, -3\pi, \ldots, -(2n+1)\pi$$

Find the location of first V_{min} and first V_{max}?



$$d_{max} = 0.5\lambda - 0.287\lambda + 0.25\lambda = 0.463\lambda$$

$$d_{min} = 0.5\lambda - 0.287\lambda = 0.213\lambda$$

Admittance (Y) Calculations

- Note:

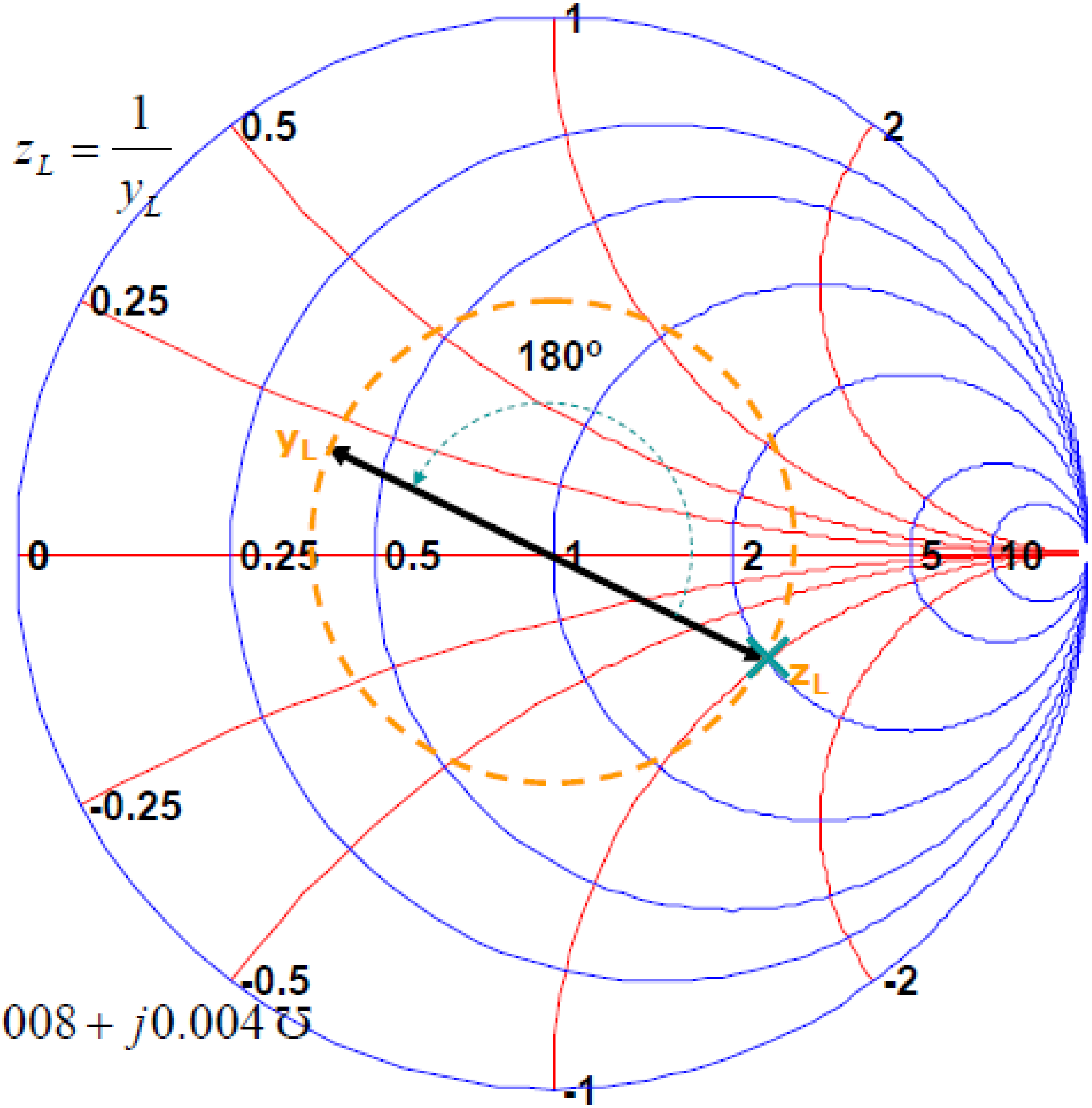
$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{z_L - 1}{z_L + 1} \rightarrow z_L = \frac{1}{y_L}$$

$$\Gamma = \frac{\frac{1}{y_L} - 1}{\frac{1}{y_L} + 1} = \frac{1 - y_L}{1 + y_L}$$

$$\Gamma = -\left(\frac{y_L - 1}{y_L + 1}\right)$$

$$\left(\frac{y_L - 1}{y_L + 1}\right) = -\Gamma$$

Accordingly, the location of the y_L is the 180° away from the location of the z_L.



$$y_L = 0.4 + j0.2 \rightarrow Y_L = 0.008 + j0.004 \text{ ℧}$$

Admittance (Y) Chart

As an alternative, we can continue to use the original Γ plane, and add admittance curves to the chart.

$$Y_n(z) = \left(\frac{1 + \left(-\Gamma(z)\right)}{1 - \left(-\Gamma(z)\right)} \right) = G_n(z) + jB_n(z)$$

Compare with previous Smith chart derivation, which started with this equation:

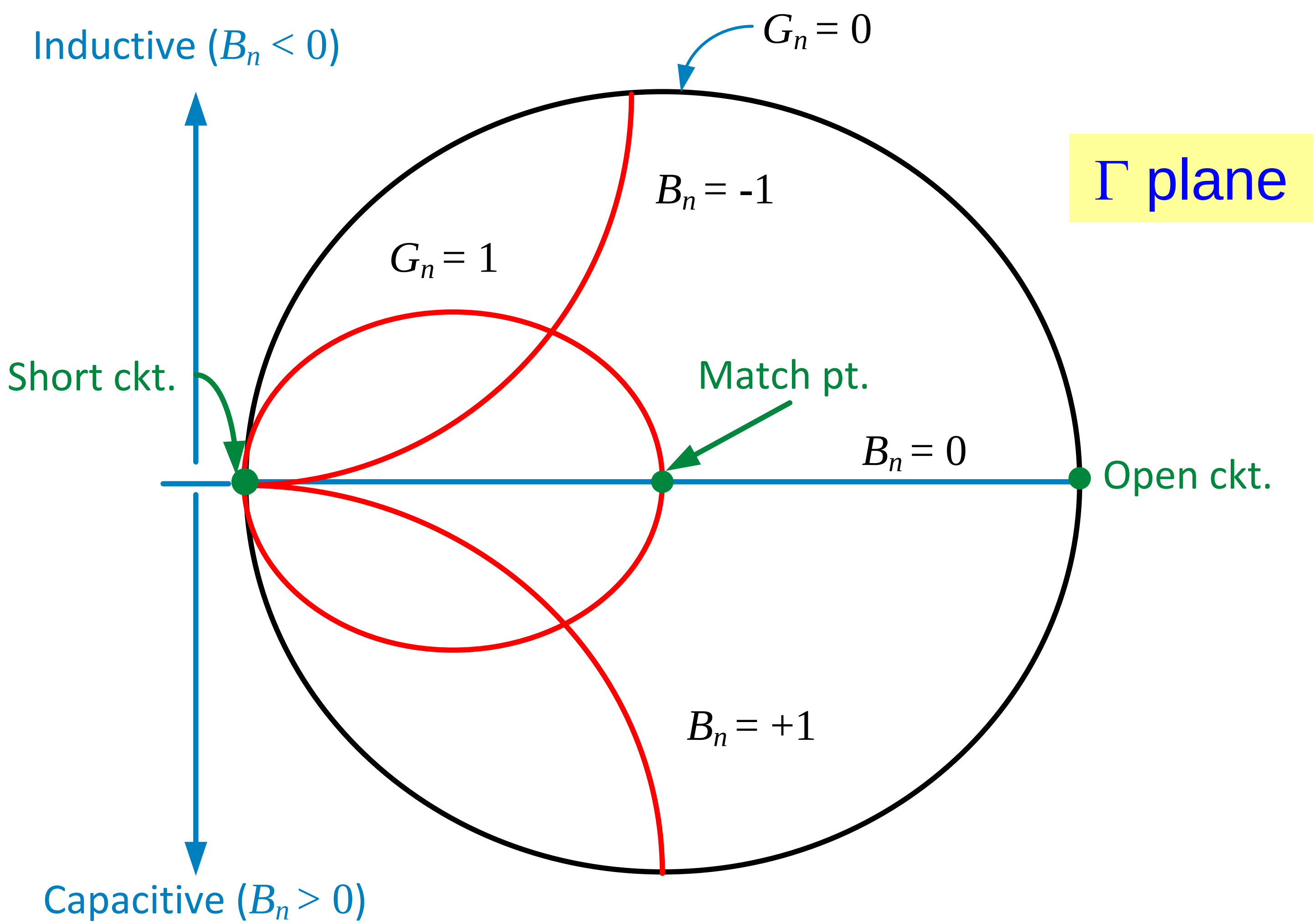
$$Z_n(z) = \left(\frac{1 + \left(\Gamma(z)\right)}{1 - \left(\Gamma(z)\right)} \right) = R_n(z) + jX_n(z)$$

If $(R_n\ X_n) = (a, b)$ is some point on the Smith chart corresponding to $\Gamma = \Gamma_0$,
Then $(G_n\ B_n) = (a, b)$ corresponds to a point located at $\Gamma = -\Gamma_0$ (180° rotation).

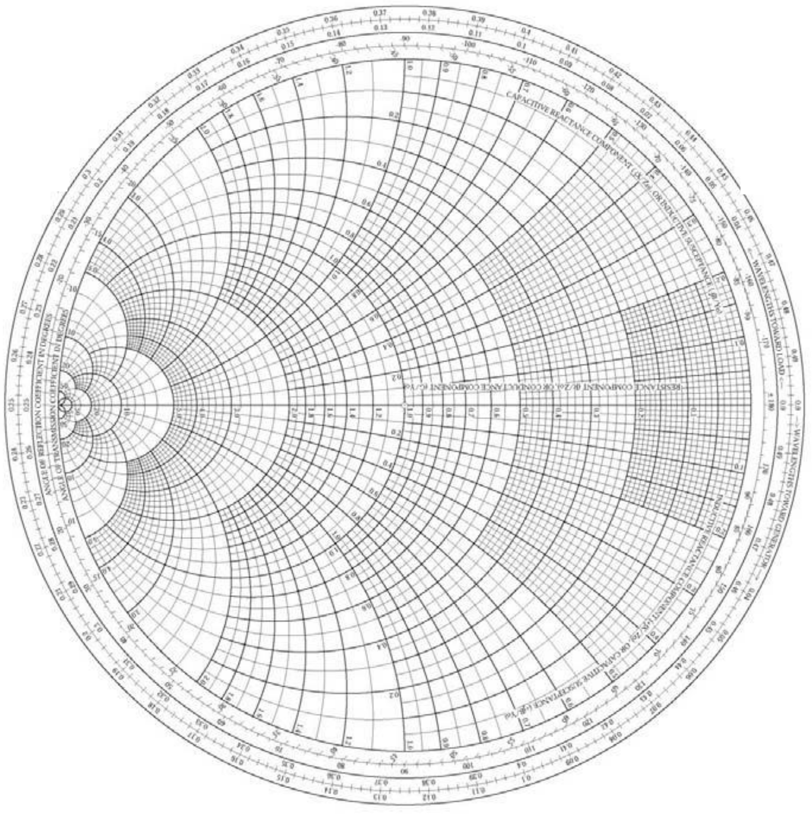
R_n circles, rotated 180°, becomes G_n circles.
X_n circles, rotated 180°, becomes B_n circles.

Side note: A 180° rotation on a Smith chart makes a normalized impedance become its reciprocal.

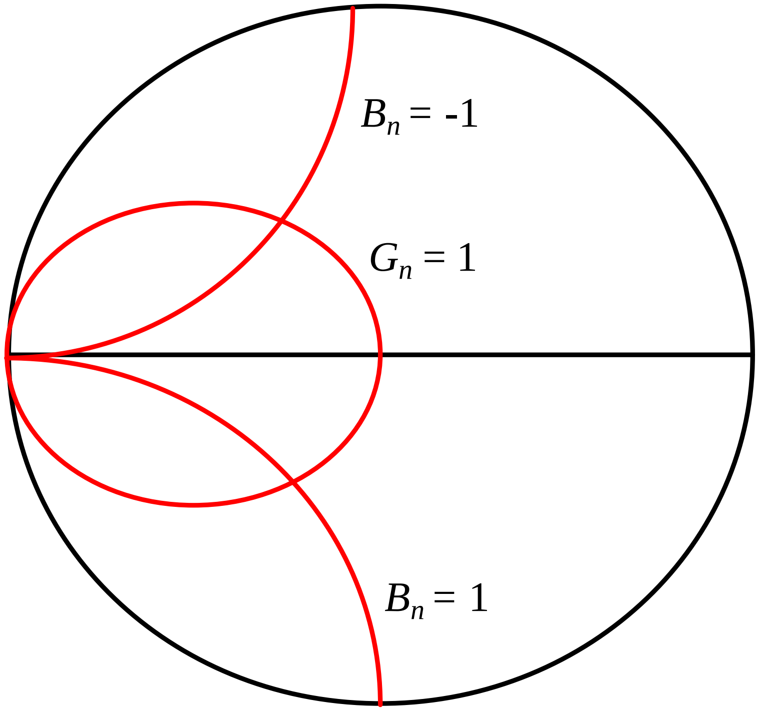
Admittance (Y) Chart (cont.)



Admittance (Y) Chart (cont.)

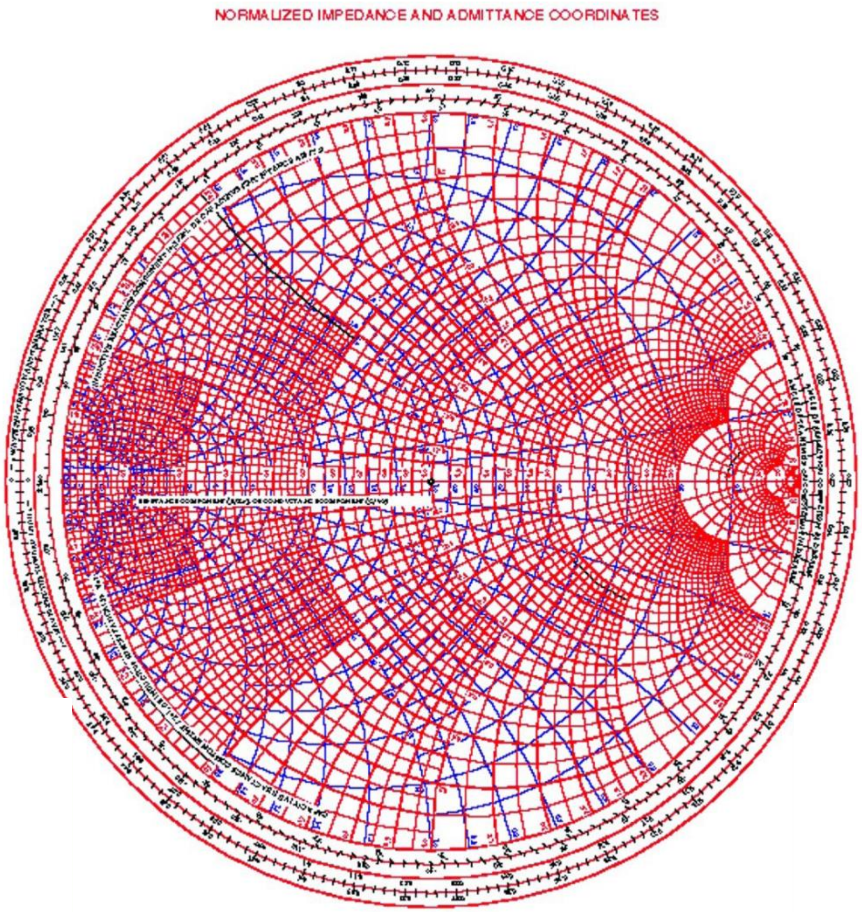


Short-hand version

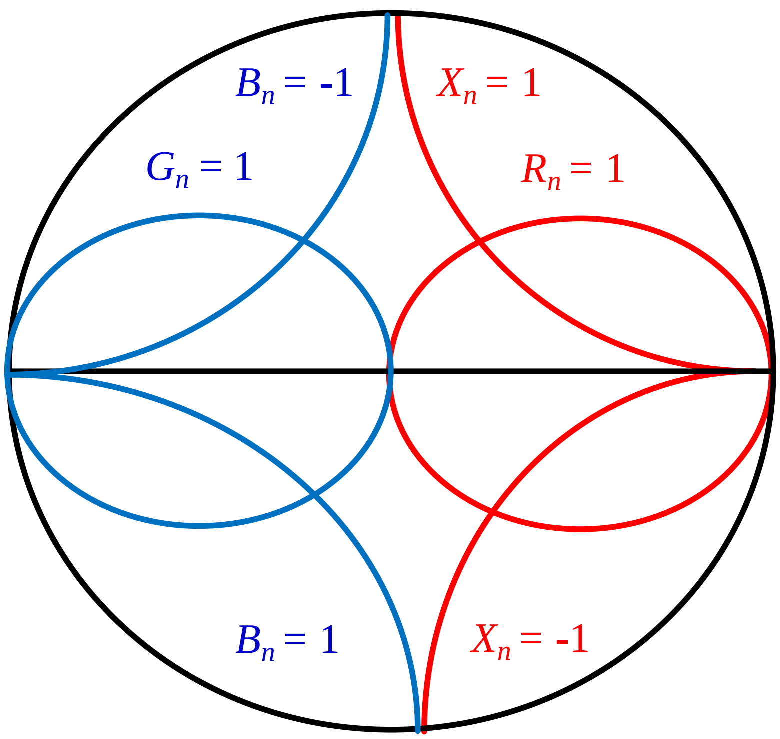


Γ plane

Impedance and Admittance (ZY) Chart



Short-hand version

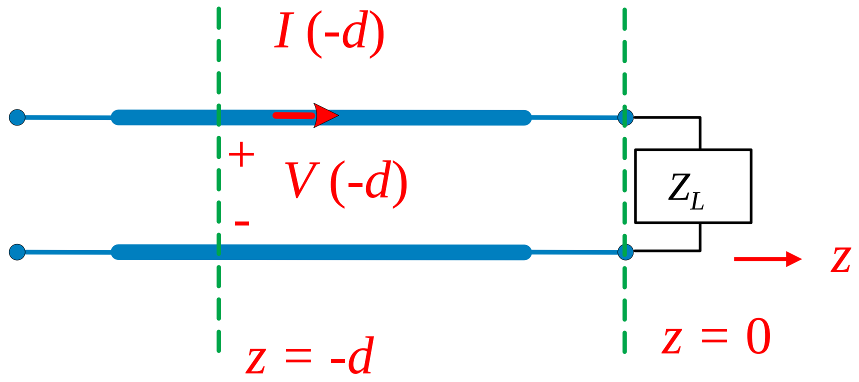


Γ plane

Example 2

$Z_0 = 50\ \Omega$

$Z_L = 100 + j50\ \Omega$



Find $Z(-d)$

at $d/\lambda_g = 1/4,\ 3/8,\ 1/2$

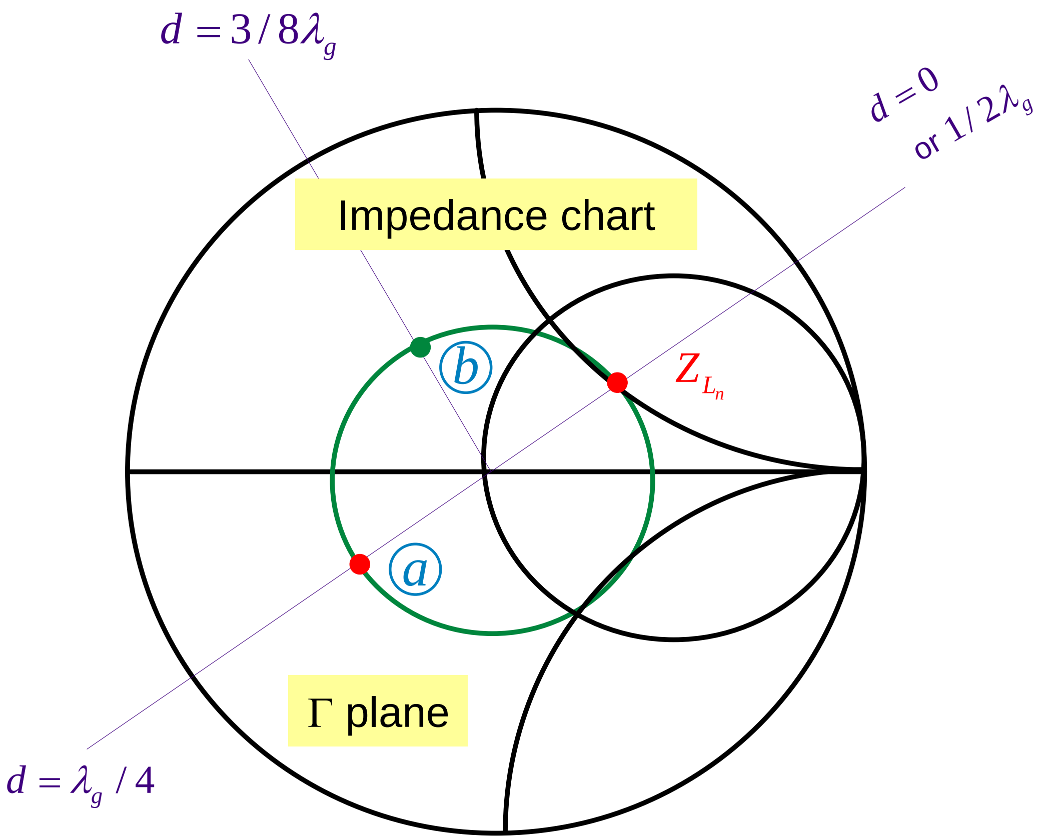
$$Z_{L,n} = \frac{Z_L}{Z_0} = 2 + j1$$

ⓐ

$d = \lambda_g / 4$

$Z_n \approx 0.4 - j0.2$

$\Rightarrow Z(-\lambda_g/4) \approx 20 - j10\Omega$



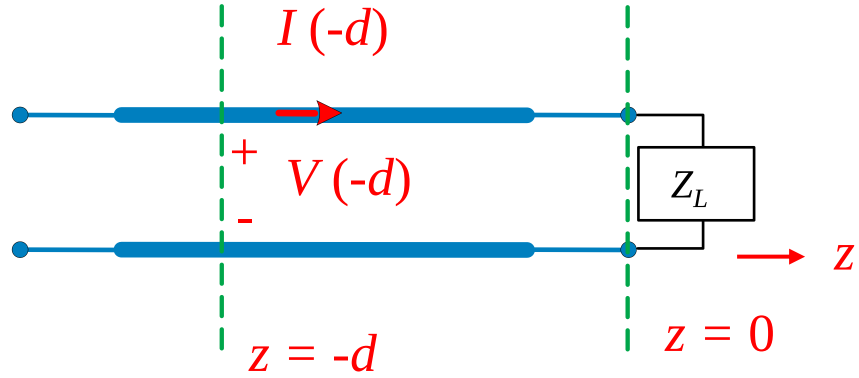
Example 2 (cont.)

ⓑ

$d = 3/8\lambda_g$

$Z_n \approx 0.5 + j0.5$

$\Rightarrow Z\left(-3/8\lambda_g\right) \approx 25 + j25\Omega$



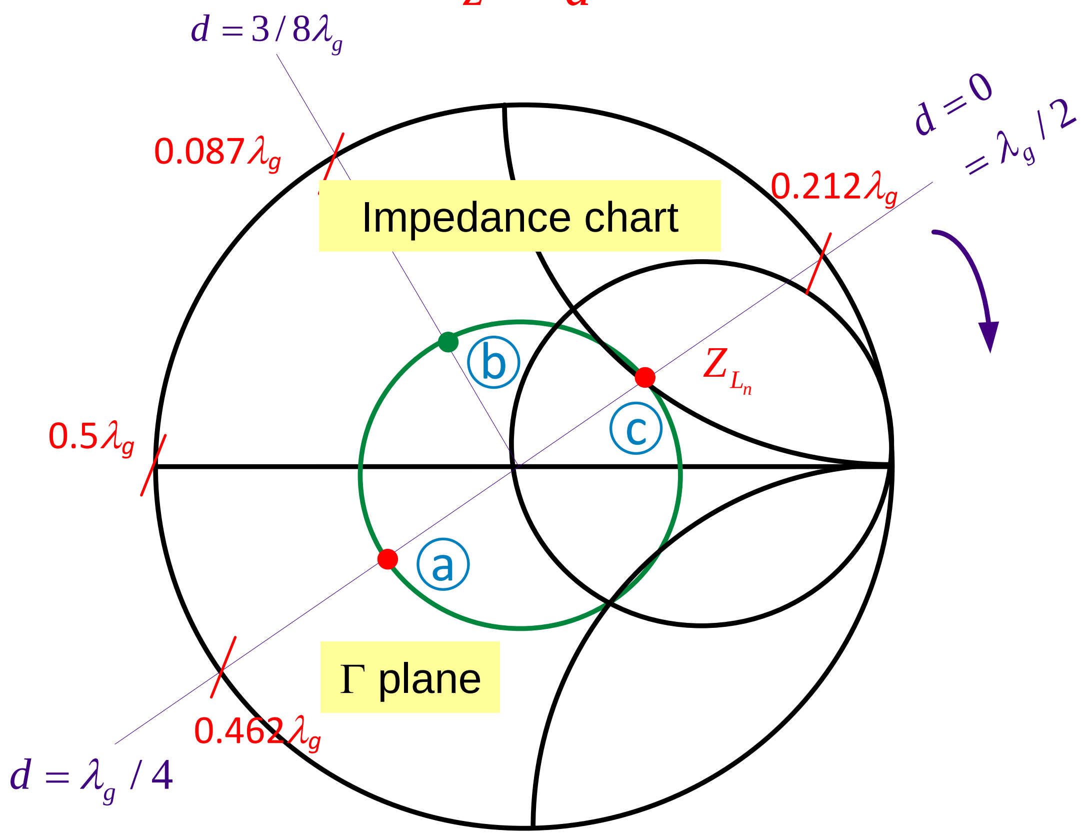
ⓒ

$d = \lambda_g / 2$

$Z_n \approx 2 + j1$

$\Rightarrow Z\left(-\lambda_g / 2\right) = 100 + j50\Omega$

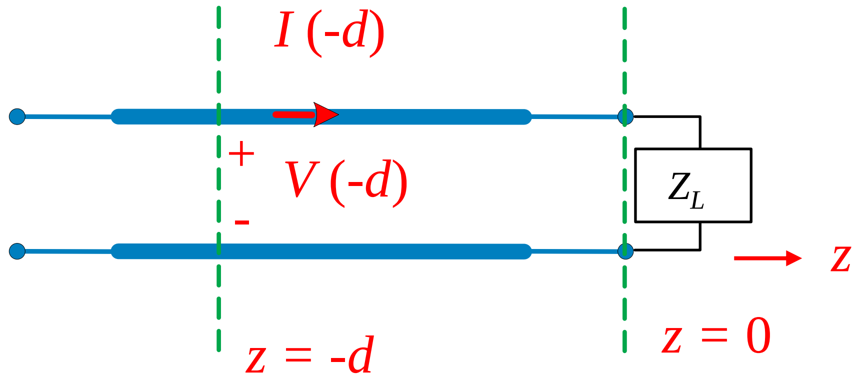
$3/8 - (0.5 - 0.212) = 0.087$



Example 3

$Z_0 = 50\ \Omega\ (Y_0 = 20\,\text{mS})$

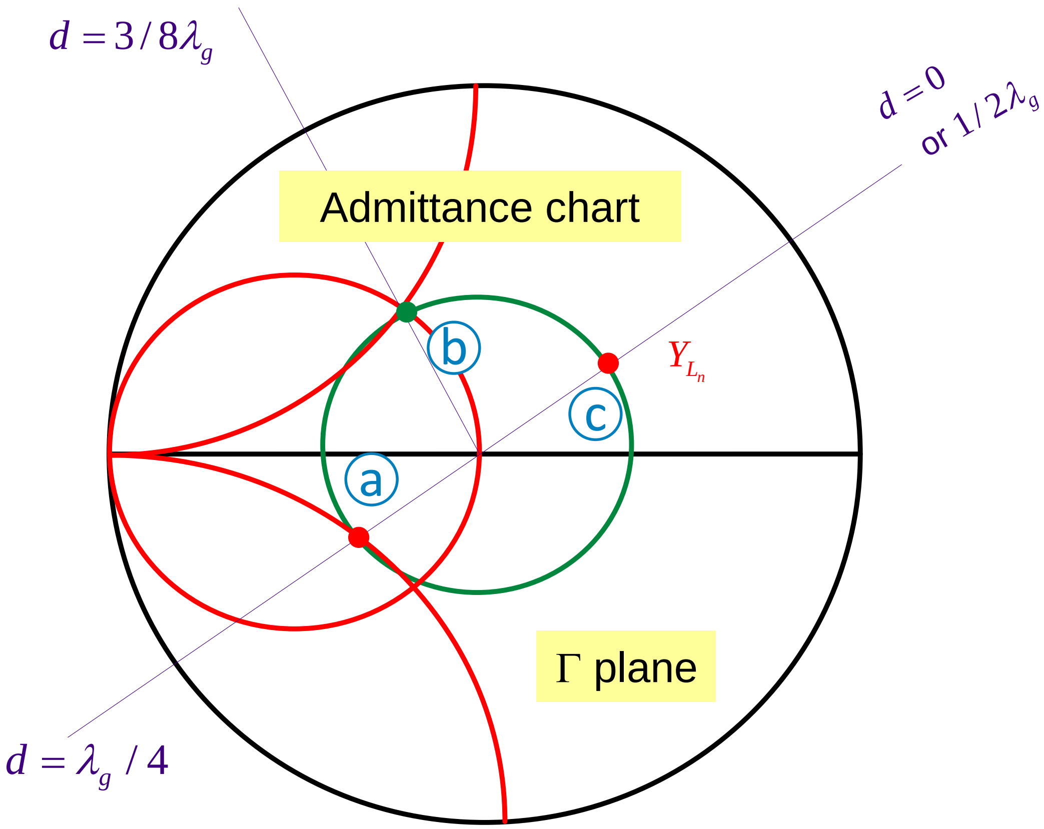
$Y_L = 8\,\text{mS} - j4\,\text{mS}$



Find $Y(-d)$

at $d/\lambda_g = 1/4,\ 3/8,\ 1/2$

$$Y_{L,n} = \frac{Y_L}{Y_0} = 0.4 - j0.2$$



ⓐ

$d = \lambda_g / 4$

$Y_n \approx 2 + j1$

$\Rightarrow Y(-\lambda_g / 4) \approx 40\,\text{mS} + j20\,\text{mS}$

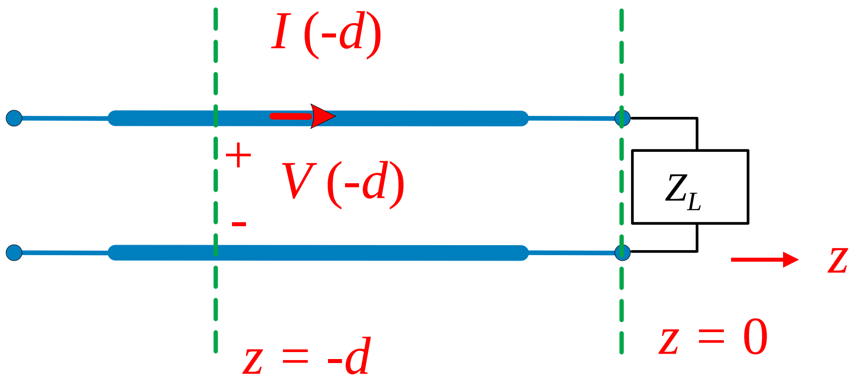
Example 3 (cont.)

ⓑ

$d = 3/8\lambda_g$

$Y_n \approx 1 - j1$

$\Rightarrow Y\left(-3/8\lambda_g\right) \approx 20\,\text{mS} - j20\,\text{mS}$

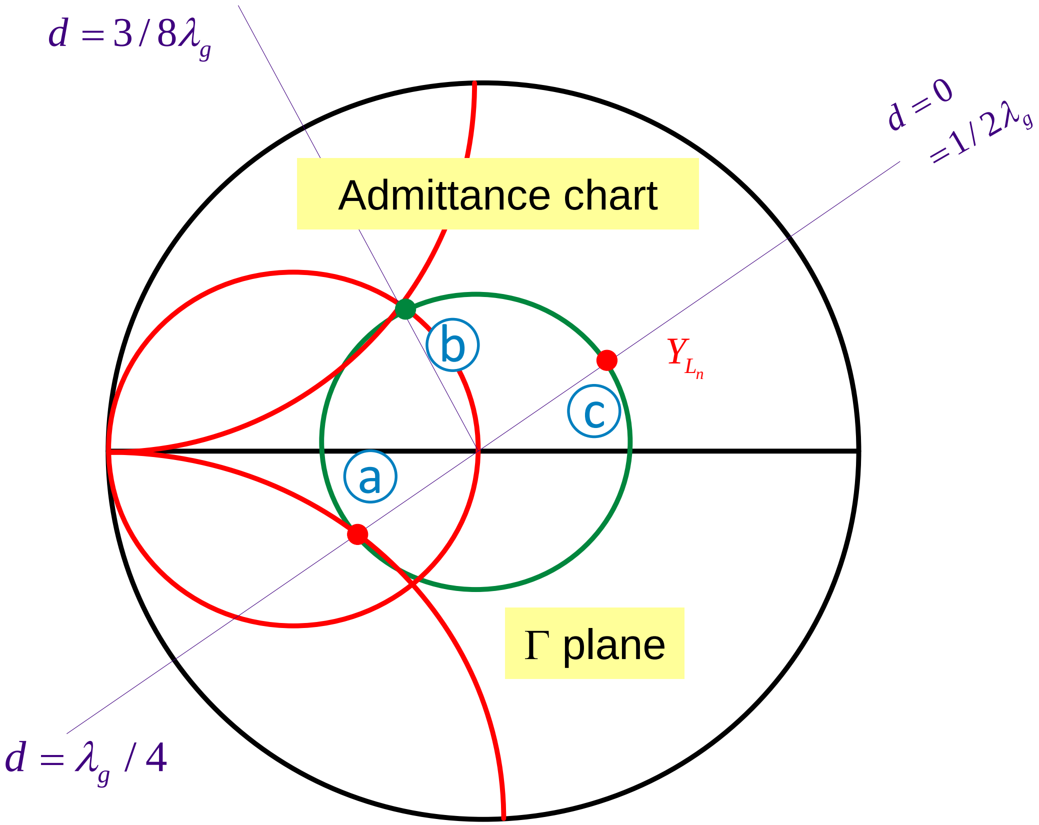


ⓒ

$d = \lambda_g / 2$

$Y_n \approx 0.4 - j0.2$

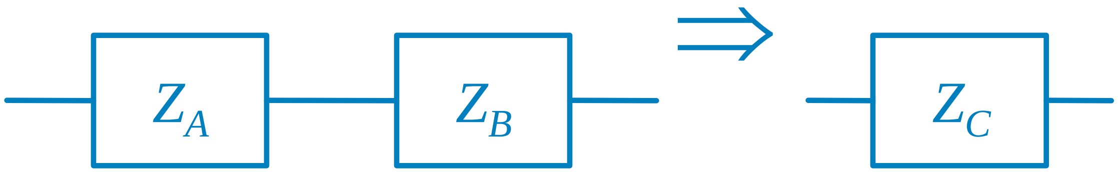
$\Rightarrow Y\left(-\lambda_g / 2\right) = 8\,\text{mS} - j4\,\text{mS}$



Which Chart to Use?

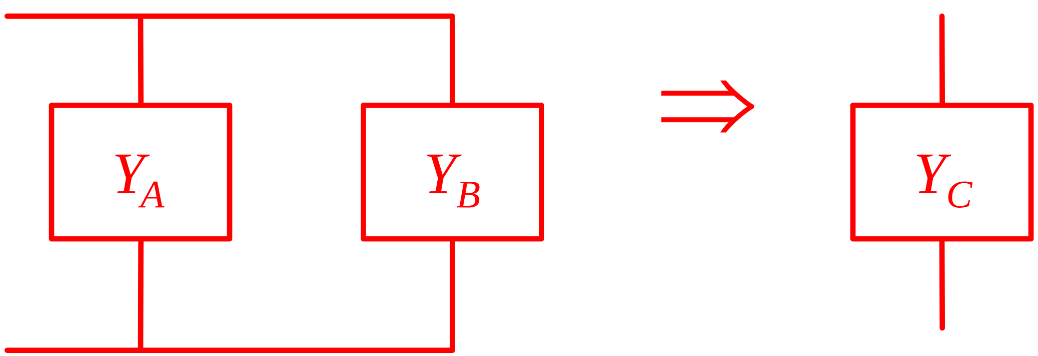
Simple answer:

* When adding elements in series use *Z*-chart



$$Z_A + Z_B = Z_C$$

* When adding elements in parallel use *Y*-chart



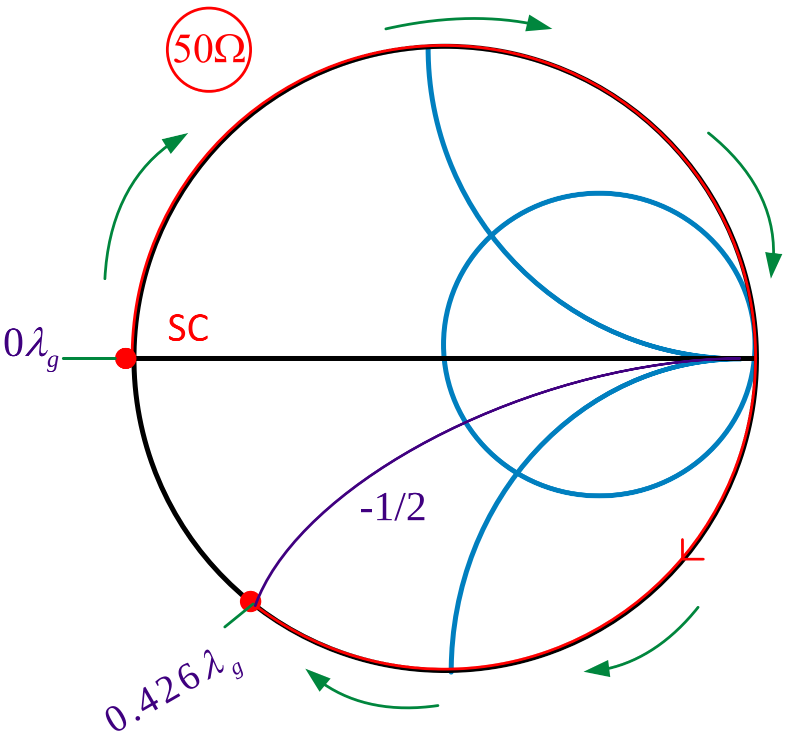
$$Y_A + Y_B = Y_C$$

Example 4

Use a short-circuited section of air-filled TEM, 50 Ω transmission line ($\beta = k_0$, $\lambda_g = \lambda_0$) to create an impedance of Z_{in} = -j25 Ω at f = 10 GHz.

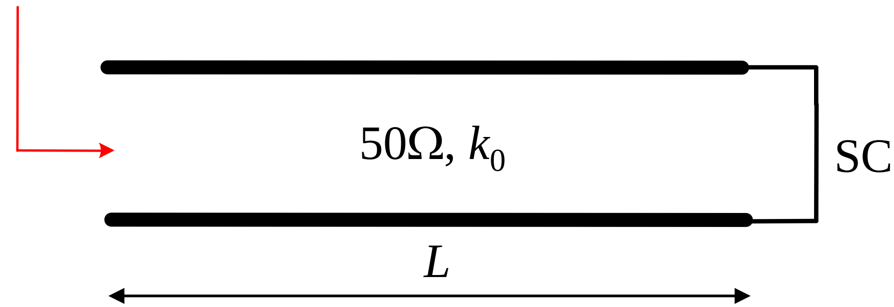
$$Z_{in,n} = -j\frac{25}{50} = -j(1/2)$$

Impedance chart



Γ plane

$$Z_{in} = -j25\Omega$$



$$L = 0.426\lambda_g - 0\lambda_g = 0.426\lambda_g$$

L = 1.28 cm

$$\lambda_0 = \frac{c}{f} = \frac{2\pi}{k_0} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}}$$

$\lambda_0 = 3.0$ cm

Electronic Smith Chart

At this link:

http://www.sss-mag.com/topten5.html

Download the following zip file:

smith_v191.zip

Extract the following files:

smith.exe mith.hlp smith.pdf

This is the application file